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Linear Equations and Inequalities in Two Variables

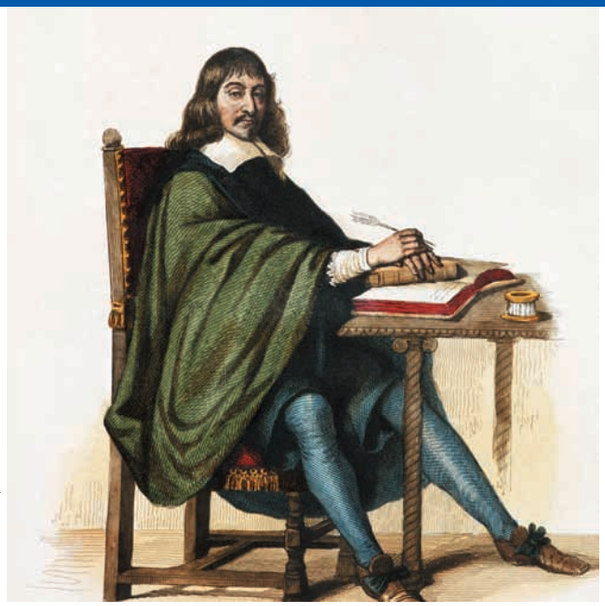
3

3.1 Rectangular Coordinate System and Linear Equations

3.2 Linear Inequalities in Two Variables

3.3 Distance and Slope

3.4 Determining the Equation of a Line



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René Descartes, a philosopher and mathematician, developed a system for locating a point on a plane. This system is our current rectangular coordinate grid used for graphing; it is named the Cartesian coordinate system.

René Descartes, a French mathematician of the 17th century, was able to transform geometric problems into an algebraic setting so that he could use the tools of algebra to solve the problems. This connecting of algebraic and geometric ideas is the foundation of a branch of mathematics called **analytic geometry**, today more commonly called **coordinate geometry**. Basically, there are two kinds of problems in coordinate geometry: Given an algebraic equation, find its geometric graph; and given a set of conditions pertaining to a geometric graph, find its algebraic equation. We discuss problems of both types in this chapter.

Video tutorials for all section learning objectives are available in a variety of delivery modes.

INTERNET PROJECT

In this chapter the rectangular coordinate system is used for graphing. Another two-dimensional coordinate system is the polar coordinate system. Conduct an Internet search to see an example of the polar coordinate system. How are the coordinates of a point determined in the polar coordinate system?

3.1 Rectangular Coordinate System and Linear Equations

OBJECTIVES

1. Find Solutions for Linear Equations in Two Variables
2. Review of the Rectangular Coordinate System
3. Graph the Solutions for Linear Equations
4. Graph Linear Equations by Finding the x and y Intercepts
5. Graph Lines Passing through the Origin, Vertical Lines, and Horizontal Lines
6. Apply Graphing to Linear Relationships
7. Introduce Graphing Utilities (Optional Exercises)

1 Find Solutions for Linear Equations in Two Variables

In this chapter we want to consider solving equations in two variables. Let's begin by considering the solutions for the equation $y = 3x + 2$. A **solution** of an equation in two variables is an ordered pair of real numbers that satisfies the equation. When using the variables x and y, we agree that the first number of an ordered pair is a value of x and the second number is a value of y. We see that $(1, 5)$ is a solution for $y = 3x + 2$ because if x is replaced by 1 and y by 5, the result is the true numerical statement $5 = 3(1) + 2$. Likewise, $(2, 8)$ is a solution because $8 = 3(2) + 2$ is a true numerical statement. We can find infinitely many pairs of real numbers that satisfy $y = 3x + 2$ by arbitrarily choosing values for x and then, for each chosen value of x, determining a corresponding value for y. Let's use a table to record some of the solutions for $y = 3x + 2$.

x value	y value determined from $y = 3x + 2$	Ordered pairs
-3	-7	$(-3, -7)$
-1	-1	$(-1, -1)$
0	2	$(0, 2)$
1	5	$(1, 5)$
2	8	$(2, 8)$
4	14	$(4, 14)$

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EXAMPLE 1 Determine some ordered-pair solutions for the equation $y = 2x - 5$ and record the values in a table.

Solution

We can start by arbitrarily choosing values for x and then determine the corresponding y value. Even though you can arbitrarily choose values for x, it is good practice to choose some negative values, zero, and some positive values.

Let $x = -4$; then, according to our equation, $y = 2(-4) - 5 = -13$.

Let $x = -1$; then, according to our equation, $y = 2(-1) - 5 = -7$.

Let $x = 0$; then, according to our equation, $y = 2(0) - 5 = -5$.

Let $x = 2$; then, according to our equation, $y = 2(2) - 5 = -1$.

Let $x = 4$; then, according to our equation, $y = 2(4) - 5 = 3$.

Organizing this information in a chart gives the following table.

x value	y value determined from $y = 2x - 5$	Ordered pair
-4	-13	$(-4, -13)$
-1	-7	$(-1, -7)$
0	-5	$(0, -5)$
2	-1	$(2, -1)$
4	3	$(4, 3)$

▼ **PRACTICE YOUR SKILL**

Determine the ordered-pair solutions for the equation $y = -2x + 4$ for the x values of -4, -2, 0, 1, and 3. Organize the information into a table. ■

$(-4, 12)$, $(-2, 8)$, $(0, 4)$, $(1, 2)$, $(3, -2)$

A table can show some of the infinite number of solutions for a linear equation in two variables, but for a visual display, solutions are plotted on a coordinate system. Let's review the rectangular coordinate system and then we can use a graph to display the solutions of an equation in two variables.

2 Review of the Rectangular Coordinate System

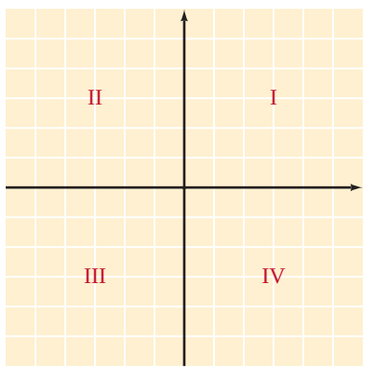

Figure 3.1

Consider two number lines, one vertical and one horizontal, perpendicular to each other at the point we associate with zero on both lines (Figure 3.1). We refer to these number lines as the **horizontal and vertical axes** or, together, as the **coordinate axes**. They partition the plane into four regions called **quadrants**. The quadrants are numbered counterclockwise from I through IV as indicated in Figure 3.1. The point of intersection of the two axes is called the **origin**.

It is now possible to set up a one-to-one correspondence between **ordered pairs** of real numbers and the points in a plane. To each ordered pair of real numbers there corresponds a unique point in the plane, and to each point in the plane there corresponds a unique ordered pair of real numbers. A part of this correspondence is illustrated in Figure 3.2. The ordered pair (3, 2) means that the point A is located three units to the right of, and two units up from, the origin. (The ordered pair (0, 0) is associated with the origin O.) The ordered pair $(-3, -5)$ means that the point D is located three units to the left and five units down from the origin.

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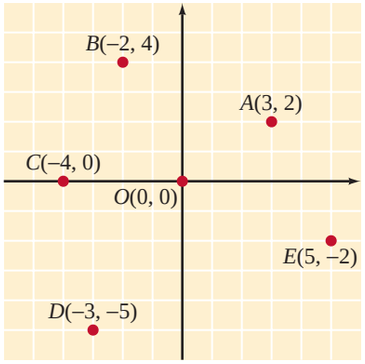
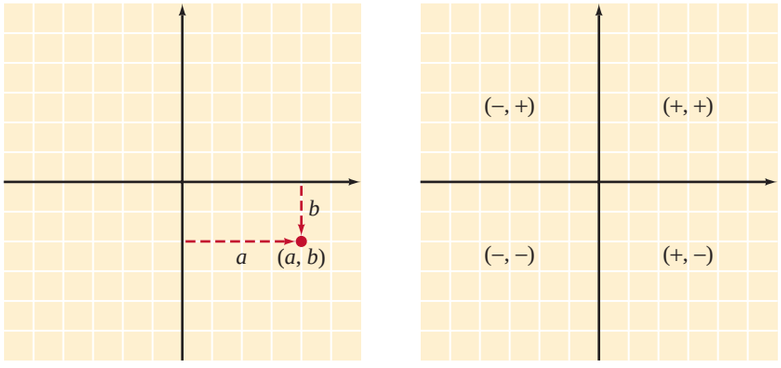


Figure 3.2

Remark: The notation $(-2, 4)$ was used earlier in this text to indicate an interval of the real number line. Now we are using the same notation to indicate an ordered pair of real numbers. This double meaning should not be confusing because the context of the material will always indicate which meaning of the notation is being used. Throughout this chapter, we will be using the ordered-pair interpretation.

In general we refer to the real numbers a and b in an ordered pair (a, b) associated with a point as the **coordinates of the point**. The first number, a, called the **abscissa**, is the directed distance of the point from the vertical axis measured parallel to the horizontal axis. The second number, b, called the **ordinate**, is the directed distance of the point from the horizontal axis measured parallel to the vertical axis (Figure 3.3a). Thus in the first quadrant all points have a positive abscissa and a positive ordinate. In the second quadrant all points have a negative abscissa and a positive ordinate. We have indicated the sign situations for all four quadrants in Figure 3.3(b). This system of associating points in a plane with pairs of real numbers is called the **rectangular coordinate system** or the **Cartesian coordinate system**.



(a) **(b)**

Figure 3.3

Historically, the rectangular coordinate system provided the basis for the development of the branch of mathematics called **analytic geometry**, or what we presently refer to as **coordinate geometry**. In this discipline, René Descartes, a French 17th-century mathematician, was able to transform geometric problems into an algebraic setting and then use the tools of algebra to solve the problems. Basically, there are two kinds of problems to solve in coordinate geometry:

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1. Given an algebraic equation, find its geometric graph.
2. Given a set of conditions pertaining to a geometric figure, find its algebraic equation.

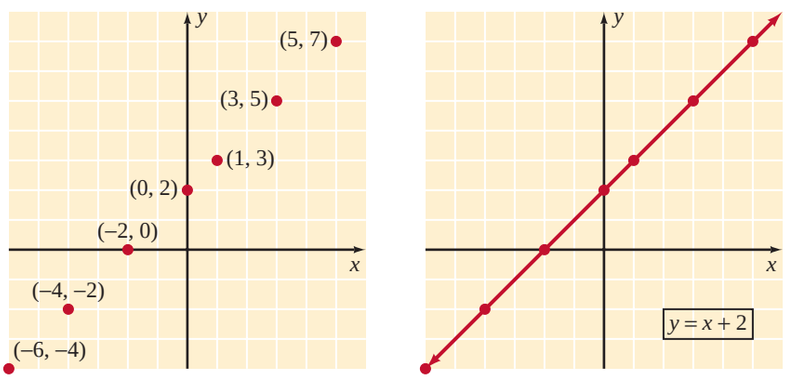
In this chapter we will discuss problems of both types. Let's start by finding the graph of an algebraic equation.

3 Graph the Solutions for Linear Equations

Let's begin by determining some solutions for the equation $y = x + 2$ and then plot the solutions on a rectangular coordinate system to produce a graph of the equation. Let's use a table to record some of the solutions.

Choose x	Determine y from $y = x + 2$	Solutions for $y = x + 2$
0	2	(0, 2)
1	3	(1, 3)
3	5	(3, 5)
5	7	(5, 7)
−2	0	(−2, 0)
−4	−2	(−4, −2)
−6	−4	(−6, −4)

We can plot the ordered pairs as points in a coordinate system and use the horizontal axis as the x axis and the vertical axis as the y axis, as in Figure 3.4(a). Connecting the points with a straight line as in Figure 3.4(b) produces a **graph of the equation** $y = x + 2$. Every point on the line has coordinates that are solutions of the equation $y = x + 2$. The graph provides a visual display of all the infinite solutions for the equation.



(a) (b)

Figure 3.4

EXAMPLE 2 Graph the equation $y = -x + 4$.

Solution

Let's begin by determining some solutions for the equation $y = -x + 4$ and then plot the solutions on a rectangular coordinate system to produce a graph of the equation. Let's use a table to record some of the solutions.

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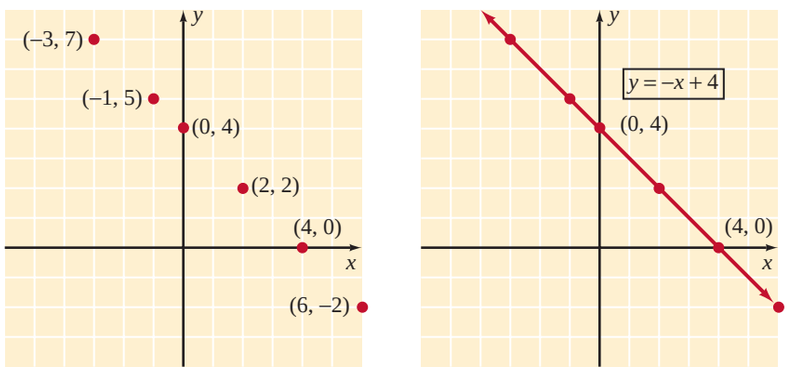
x value	y value determined from $y = -x + 4$	Ordered pairs
-3	7	$(-3, 7)$
-1	5	$(-1, 5)$
0	4	$(0, 4)$
2	2	$(2, 2)$
4	0	$(4, 0)$
6	-2	$(6, -2)$

We can plot the ordered pairs on a coordinate system as shown in Figure 3.5(a). The graph of the equation is produced by drawing a straight line through the plotted points as in Figure 3.5(b).



(a) **(b)**

Figure 3.5

▼ **PRACTICE YOUR SKILL**

Graph the equation $y = 2x - 2$. See answer section. ■

4 Graph Linear Equations by Finding the x and y Intercepts

The points (4, 0) and (0, 4) in Figure 3.5(b) are special points. They are the points of the graph that are on the coordinate axes. That is, they yield the x intercept and the y intercept of the graph. Let's define in general the *intercepts* of a graph.

The x coordinates of the points that a graph has in common with the x axis are called the ***x* intercepts** of the graph. (To compute the x intercepts, let $y = 0$ and solve for x.)

The y coordinates of the points that a graph has in common with the y axis are called the ***y* intercepts** of the graph. (To compute the y intercepts, let $x = 0$ and solve for y.)

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It is advantageous to be able to recognize the kind of graph that a certain type of equation produces. For example, if we recognize that the graph of $3x + 2y = 12$ is a straight line, then it becomes a simple matter to find two points and sketch the line. Let's pursue the graphing of straight lines in a little more detail.

In general, any equation of the form $Ax + By = C$, where A, B, and C are constants (A and B not both zero) and x and y are variables, is a **linear equation**, and its graph is a straight line. Two points of clarification about this description of a linear equation should be made. First, the choice of x and y for variables is arbitrary. Any two letters could be used to represent the variables. For example, an equation such as $3r + 2s = 9$ can be considered a linear equation in two variables. So that we are not constantly changing the labeling of the coordinate axes when graphing equations, however, it is much easier to use the same two variables in all equations. Thus we will go along with convention and use x and y as variables. Second, the phrase "any equation of the form $Ax + By = C$" technically means "any equation of the form $Ax + By = C$ or equivalent to that form." For example, the equation $y = 2x - 1$ is equivalent to $-2x + y = -1$ and thus is linear and produces a straight-line graph.

The knowledge that any equation of the form $Ax + By = C$ produces a straight-line graph, along with the fact that two points determine a straight line, makes graphing linear equations a simple process. We merely find two solutions (such as the intercepts), plot the corresponding points, and connect the points with a straight line. It is usually wise to find a third point as a check point. Let's consider an example.

EXAMPLE 3 Graph $3x - 2y = 12$.

Solution

First, let's find the intercepts. Let $x = 0$; then

$$\begin{aligned} 3(0) - 2y &= 12 \\ -2y &= 12 \\ y &= -6 \end{aligned}$$

Thus $(0, -6)$ is a solution. Let $y = 0$; then

$$\begin{aligned} 3x - 2(0) &= 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

Thus $(4, 0)$ is a solution. Now let's find a third point to serve as a check point. Let $x = 2$; then

$$\begin{aligned} 3(2) - 2y &= 12 \\ 6 - 2y &= 12 \\ -2y &= 6 \\ y &= -3 \end{aligned}$$

Thus $(2, -3)$ is a solution. Plot the points associated with these three solutions and connect them with a straight line to produce the graph of $3x - 2y = 12$ in Figure 3.6.

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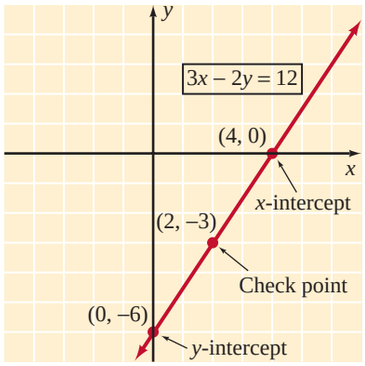

Figure 3.6

▼ PRACTICE YOUR SKILL

Graph $3x - y = 6$. See answer section. ■

Let's review our approach to Example 3. Note that we did not solve the equation for y in terms of x or for x in terms of y. Because we know the graph is a straight line, there is no need for any extensive table of values. Furthermore, the solution $(2, -3)$ served as a check point. If it had not been on the line determined by the two intercepts, then we would have known that an error had been made.

EXAMPLE 4

Graph $2x + 3y = 7$.

Solution

Without showing all of our work, the following table indicates the intercepts and a check point. The points from the table are plotted, and the graph of $2x + 3y = 7$ is shown in Figure 3.7.

x	y	
0	$\frac{7}{3}$	
$\frac{7}{2}$	0	Intercepts
2	1	Check point

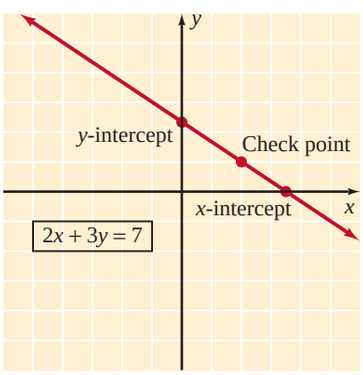

Figure 3.7

▼ PRACTICE YOUR SKILL

Graph $x + 2y = 3$. See answer section. ■

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5 Graph Lines Passing through the Origin, Vertical Lines, and Horizontal Lines

It is helpful to recognize some *special* straight lines. For example, the graph of any equation of the form $Ax + By = C$, where $C = 0$ (the constant term is zero), is a straight line that contains the origin. Let's consider an example.

EXAMPLE 5 Graph $y = 2x$.

Solution

Obviously $(0, 0)$ is a solution. (Also, notice that $y = 2x$ is equivalent to $-2x + y = 0$; thus it fits the condition $Ax + By = C$, where $C = 0$.) Because both the x intercept and the y intercept are determined by the point $(0, 0)$, another point is necessary to determine the line. Then a third point should be found as a check point. The graph of $y = 2x$ is shown in Figure 3.8.

x	y	
0	0	Intercepts
2	4	Additional point
-1	-2	Check point

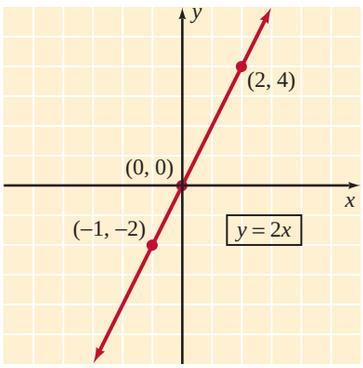

Figure 3.8

▼ **PRACTICE YOUR SKILL**

Graph $y = -3x$. See answer section. ■

EXAMPLE 6 Graph $x = 2$.

Solution

Because we are considering linear equations in *two variables*, the equation $x = 2$ is equivalent to $x + 0(y) = 2$. Now we can see that any value of y can be used, but the x value must always be 2. Therefore, some of the solutions are $(2, 0)$, $(2, 1)$, $(2, 2)$, $(2, -1)$, and $(2, -2)$. The graph of all solutions of $x = 2$ is the vertical line in Figure 3.9.

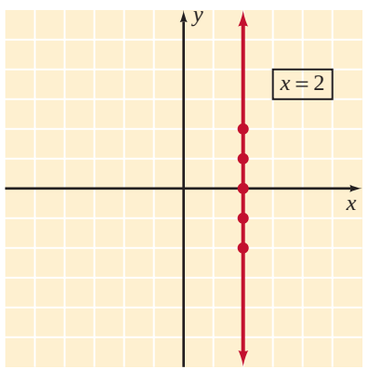

Figure 3.9

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▼ PRACTICE YOUR SKILL

Graph $x = -3$. See answer section. ■

EXAMPLE 7 Graph $y = -3$.

Solution

The equation $y = -3$ is equivalent to $0(x) + y = -3$. Thus any value of x can be used, but the value of y must be -3. Some solutions are $(0, -3)$, $(1, -3)$, $(2, -3)$, $(-1, -3)$, and $(-2, -3)$. The graph of $y = -3$ is the horizontal line in Figure 3.10.

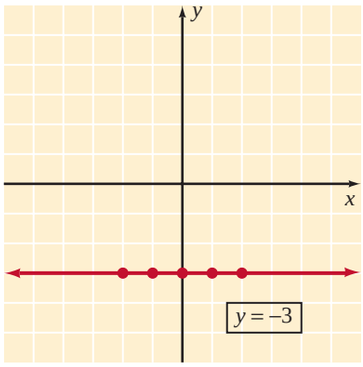

Figure 3.10

▼ PRACTICE YOUR SKILL

Graph $y = 4$. See answer section. ■

In general, the graph of any equation of the form $Ax + By = C$, where $A = 0$ or $B = 0$ (not both), is a line parallel to one of the axes. More specifically, any equation of the form $x = a$, where a is a constant, is a line parallel to the y axis that has an x intercept of a. Any equation of the form $y = b$, where b is a constant, is a line parallel to the x axis that has a y intercept of b.

6 Apply Graphing to Linear Relationships

There are numerous applications of linear relationships. For example, suppose that a retailer has a number of items that she wants to sell at a profit of 30% of the cost of each item. If we let s represent the selling price and c the cost of each item, then the equation

$$s = c + 0.3c = 1.3c$$

can be used to determine the selling price of each item based on the cost of the item. In other words, if the cost of an item is \$4.50, then it should be sold for $s = (1.3)(4.5) = \$5.85$.

The equation $s = 1.3c$ can be used to determine the following table of values. Reading from the table, we see that if the cost of an item is \$15, then it should be sold for \$19.50 in order to yield a profit of 30% of the cost. Furthermore, because this is a linear relationship, we can obtain exact values between values given in the table.

c	1	5	10	15	20
s	1.3	6.5	13	19.5	26

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For example, a c value of 12.5 is halfway between c values of 10 and 15, so the corresponding s value is halfway between the s values of 13 and 19.5. Therefore, a c value of 12.5 produces an s value of

$$s = 13 + \frac{1}{2}(19.5 - 13) = 16.25$$

Thus, if the cost of an item is \$12.50, it should be sold for \$16.25.

Now let's graph this linear relationship. We can label the horizontal axis c, label the vertical axis s, and use the origin along with one ordered pair from the table to produce the straight-line graph in Figure 3.11. (Because of the type of application, we use only nonnegative values for c and s.)

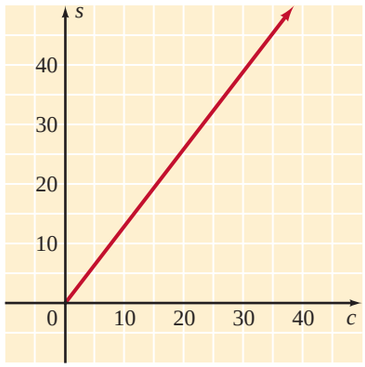

Figure 3.11

From the graph we can approximate s values on the basis of given c values. For example, if $c = 30$, then by reading up from 30 on the c axis to the line and then across to the s axis, we see that s is a little less than 40. (An exact s value of 39 is obtained by using the equation $s = 1.3c$.)

Many formulas that are used in various applications are linear equations in two variables. For example, the formula $C = \frac{5}{9}(F - 32)$, which is used to convert temperatures from the Fahrenheit scale to the Celsius scale, is a linear relationship. Using this equation, we can determine that 14°F is equivalent to $C = \frac{5}{9}(14 - 32) = \frac{5}{9}(-18) = -10$°C. Let's use the equation $C = \frac{5}{9}(F - 32)$ to complete the following table.

F	−22	−13	5	32	50	68	86
C	−30	−25	−15	0	10	20	30

Reading from the table, we see, for example, that −13°F = −25°C and 68°F = 20°C.

To graph the equation $C = \frac{5}{9}(F - 32)$ we can label the horizontal axis F, label the vertical axis C, and plot two ordered pairs (F, C) from the table. Figure 3.12 shows the graph of the equation.

From the graph we can approximate C values on the basis of given F values. For example, if F = 80°, then by reading up from 80 on the F axis to the line and then across to the C axis, we see that C is approximately 25°. Likewise, we can obtain approximate F values on the basis of given C values. For example, if C = −25°, then by

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reading across from −25 on the C axis to the line and then up to the F axis, we see that F is approximately −15°.

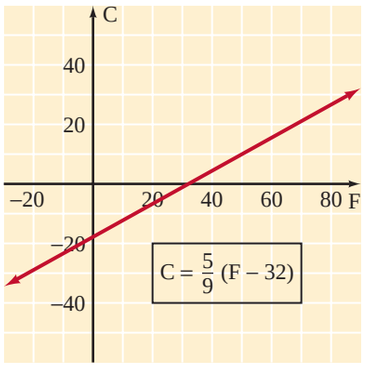
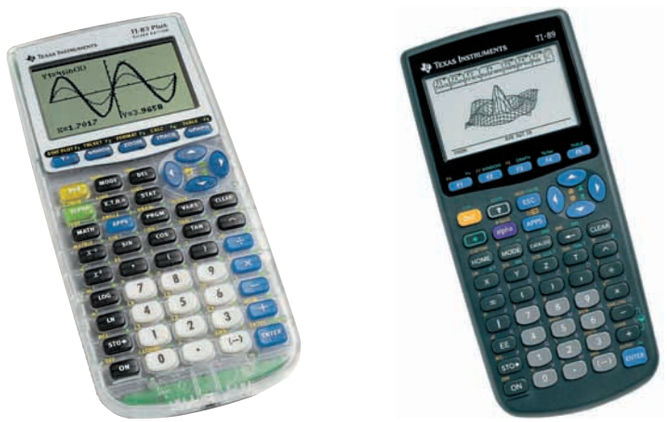


Figure 3.12

7 Introduce Graphing Utilities

The term **graphing utility** is used in current literature to refer to either a graphing calculator (see Figure 3.13) or a computer with a graphing software package. (We will frequently use the phrase *use a graphing calculator* to mean "use a graphing calculator or a computer with the appropriate software.")

These devices have a large range of capabilities that enable the user not only to obtain a quick sketch of a graph but also to study various characteristics of it, such as the x intercepts, y intercepts, and turning points of a curve. We will introduce some of these features of graphing utilities as we need them in the text. Because there are so many different types of graphing utilities available, we will use mostly generic terminology and let you consult your user's manual for specific key-punching instructions. We urge you to study the graphing utility examples in this text even if you do not have access to a graphing calculator or a computer. The examples were chosen to reinforce concepts under discussion.



Courtesy Texas Instruments

Figure 3.13

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EXAMPLE 8 Use a graphing utility to obtain a graph of the line $2.1x + 5.3y = 7.9$.

Solution

First, let's solve the equation for y in terms of x.

$$2.1x + 5.3y = 7.9$$

$$5.3y = 7.9 - 2.1x$$

$$y = \frac{7.9 - 2.1x}{5.3}$$

Now we can enter the expression $\frac{7.9 - 2.1x}{5.3}$ for Y_1 and obtain the graph as shown in Figure 3.14.

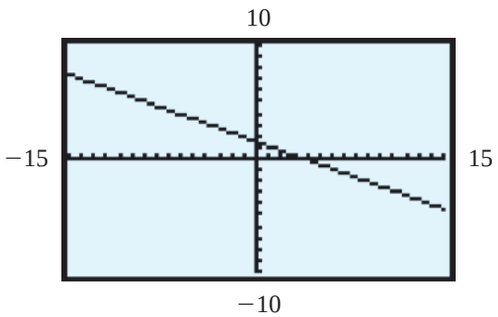

Figure 3.14

▼ PRACTICE YOUR SKILL

Use a graphing utility to obtain a graph of the line $3.4x - 2.5y = 6.8$. ■

See answer section.

CONCEPT QUIZ For Problems 1–10, answer true or false.

1. In a rectangular coordinate system, the coordinate axes partition the plane into four parts called quadrants.
2. Quadrants are named with Roman numerals and are numbered clockwise.
3. The real numbers in an ordered pair are referred to as the coordinates of the point.
4. If the abscissa of an ordered pair is negative, then the point is in either the 3rd or 4th quadrant.
5. The equation $y = x + 3$ has an infinite number of ordered pairs that satisfy the equation.
6. The graph of $y = x^2$ is a straight line.
7. The y intercept of the graph of $3x + 4y = -4$ is -4.
8. The graph of $y = 4$ is a vertical line.
9. The graph of $x = 4$ has an x intercept of 4.
10. The graph of every linear equation has a y intercept.

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Problem Set 3.1

1 Find Solutions for Linear Equations in Two Variables

For Problems 1–4, determine which of the ordered pairs are solutions to the given equation.

1. $y = 3x - 2$ $(2, 4), (-1, -5), (0, 1)$ (2, 4), (−1, −5)

2. $y = 2x + 3$ $(2, 5), (1, 5), (-1, 1)$ (1, 5), (−1, 1)

3. $2x + y = 6$ $(-2, 10), (-1, 5), (3, 0)$ (−2,10), (3, 0)

4. $-3x + 2y = 2$ $\left(3, \frac{11}{2}\right), (-2, -2) \left(-1, -\frac{1}{2}\right)$
$\left(3, \frac{11}{2}\right)$ (−2, −2) $\left(-1, -\frac{1}{2}\right)$

3 Graph the Solutions for Linear Equations

For Problems 5–8, complete the table of values for the equation and graph the equation. See answer section.

5. $y = -x + 3$

x	−2	−1	0	4
y	5	4	3	−1

6. $y = 2x - 1$

x	−3	−1	0	2
y	−7	−3	−1	3

7. $2x - y = 6$

x	−2	0	2	4
y	−10	−6	−2	2

8. $2x - 3y = -6$

x	−3	0	2	3
y	0	2	$\frac{10}{3}$	4

4 Graph Linear Equations by Finding the *x* and *y* Intercepts

For Problems 9–28, graph each of the linear equations by finding the x and y intercepts. See answer section.

▶ **9.** $x + 2y = 4$

10. $2x + y = 6$

11. $2x - y = 2$

12. $3x - y = 3$

▶**13.** $3x + 2y = 6$

14. $2x + 3y = 6$

▶**15.** $5x - 4y = 20$

16. $4x - 3y = -12$

▶**17.** $x + 4y = -6$

18. $5x + y = -2$

▶**19.** $-x - 2y = 3$

20. $-3x - 2y = 12$

▶**21.** $y = x + 3$

22. $y = x - 1$

▶**23.** $y = -2x - 1$

24. $y = 4x + 3$

▶**25.** $y = \frac{1}{2}x + \frac{2}{3}$

26. $y = \frac{2}{3}x - \frac{3}{4}$

▶**27.** $-3y = -x + 3$

28. $2y = x - 2$

5 Graph Lines Passing through the Origin, Vertical Lines, and Horizontal Lines

For Problems 29–40, graph each of the linear equations. See answer section.

▶**29.** $y = -x$

30. $y = x$

▶**31.** $y = 3x$

32. $y = -4x$

▶**33.** $2x - 3y = 0$

34. $3x + 4y = 0$

▶**35.** $x = 0$

36. $y = 0$

▶**37.** $y = 2$

38. $x = -3$

▶**39.** $x = -4$

40. $y = -1$

6 Apply Graphing to Linear Relationships

▶**41.** **(a)** Digital Solutions charges for help-desk services according to the equation $c = 0.25m + 10$, where c represents the cost in dollars and m represents the minutes of service. Complete the following table.

m	5	10	15	20	30	60
c	11.25	12.50	13.75	15.00	17.50	25.00

(b) Label the horizontal axis m and the vertical axis c, and graph the equation $c = 0.25m + 10$ for nonnegative values of m.

(c) Use the graph from part (b) to approximate values for c when $m = 25$, 40, and 45.

(d) Check the accuracy of your readings from the graph in part (c) by using the equation $c = 0.25m + 10$.

▶**42.** **(a)** The equation $F = \frac{9}{5}C + 32$ can be used to convert from degrees Celsius to degrees Fahrenheit. Complete the following table.

C	0	5	10	15	20	−5	−10	−15	−20	−25
F	32	41	50	59	68	23	14	5	−4	−13

(b) Graph the equation $F = \frac{9}{5}C + 32$.

(c) Use your graph from part (b) to approximate values for F when C = 25°, 30°, −30°, and −40°.

(d) Check the accuracy of your readings from the graph in part (c) by using the equation $F = \frac{9}{5}C + 32$.

77, 86, −22, −40

43. **(a)** A doctor's office wants to chart and graph the linear relationship between the hemoglobin A1c reading and the average blood glucose level. The equation $G = 30h - 60$ describes the relationship, where h is the hemoglobin A1c reading and G is the average blood glucose reading. Complete this chart of values:

Hemoglobin A1c, h	6.0	6.5	7.0	8.0	8.5	9.0	10.0
Blood glucose, G	120	135	150	180	195	210	240

▶Blue arrows indicate Enhanced WebAssign problems.

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(b) Label the horizontal axis h and the vertical axis G, then graph the equation $G = 30h - 60$ for h values between 4.0 and 12.0. See answer section.

(c) Use the graph from part (b) to approximate values for G when $h = 5.5$ and 7.5.

(d) Check the accuracy of your readings from the graph in part (c) by using the equation $G = 30h - 60$.

$G = 105$, $G = 165$

44. Suppose that the daily profit from an ice cream stand is given by the equation $p = 2n - 4$, where n represents the gallons of ice cream mix used in a day and p represents the dollars of profit. Label the horizontal axis n and the vertical axis p, and graph the equation $p = 2n - 4$ for nonnegative values of n.
See answer section.

45. The cost (c) of playing an online computer game for a time (t) in hours is given by the equation $c = 3t + 5$. Label the horizontal axis t and the vertical axis c, and graph the equation for nonnegative values of t.
See answer section.

46. The area of a sidewalk whose width is fixed at 3 feet can be given by the equation $A = 3l$, where A represents the area in square feet and l represents the length in feet. Label the horizontal axis l and the vertical axis A, and graph the equation $A = 3l$ for nonnegative values of l.
See answer section.

47. An online grocery store charges for delivery based on the equation $C = 0.30p$, where C represents the cost in dollars and p represents the weight of the groceries in pounds. Label the horizontal axis p and the vertical axis C, and graph the equation $C = 0.30p$ for nonnegative values of p.
See answer section.

THOUGHTS INTO WORDS

48. How do we know that the graph of $y = -3x$ is a straight line that contains the origin?

49. How do we know that the graphs of $2x - 3y = 6$ and $-2x + 3y = -6$ are the same line?

50. What is the graph of the conjunction $x = 2$ and $y = 4$? What is the graph of the disjunction $x = 2$ or $y = 4$? Explain your answers.

51. Your friend claims that the graph of the equation $x = 2$ is the point $(2, 0)$. How do you react to this claim?

FURTHER INVESTIGATIONS

From our work with absolute value, we know that $|x + y| = 1$ is equivalent to $x + y = 1$ or $x + y = -1$. Therefore, the graph of $|x + y| = 1$ consists of the two lines $x + y = 1$ and $x + y = -1$. Graph each of the following. See answer section.

52. $|x + y| = 1$

53. $|x - y| = 4$

54. $|2x - y| = 4$

55. $|3x + 2y| = 6$

GRAPHING CALCULATOR ACTIVITIES

This is the first of many appearances of a group of problems called graphing calculator activities. These problems are specifically designed for those of you who have access to a graphing calculator or a computer with an appropriate software package. Within the framework of these problems, you will be given the opportunity to reinforce concepts we discussed in the text; lay groundwork for concepts we will introduce later in the text; predict shapes and locations of graphs on the basis of your previous graphing experiences; solve problems that are unreasonable or perhaps impossible to solve without a graphing utility; and in general become familiar with the capabilities and limitations of your graphing utility.

56. **(a)** Graph $y = 3x + 4$, $y = 2x + 4$, $y = -4x + 4$, and $y = -2x + 4$ on the same set of axes.

(b) Graph $y = \frac{1}{2}x - 3$, $y = 5x - 3$, $y = 0.1x - 3$, and $y = -7x - 3$ on the same set of axes.

(c) What characteristic do all lines of the form $y = ax + 2$ (where a is any real number) share?

57. **(a)** Graph $y = 2x - 3$, $y = 2x + 3$, $y = 2x - 6$, and $y = 2x + 5$ on the same set of axes.

(b) Graph $y = -3x + 1$, $y = -3x + 4$, $y = -3x - 2$, and $y = -3x - 5$ on the same set of axes.

(c) Graph $y = \frac{1}{2}x + 3$, $y = \frac{1}{2}x - 4$, $y = \frac{1}{2}x + 5$, and $y = \frac{1}{2}x - 2$ on the same set of axes.

(d) What relationship exists among all lines of the form $y = 3x + b$, where b is any real number?

58. **(a)** Graph $2x + 3y = 4$, $2x + 3y = -6$, $4x - 6y = 7$, and $8x + 12y = -1$ on the same set of axes.

(b) Graph $5x - 2y = 4$, $5x - 2y = -3$, $10x - 4y = 3$, and $15x - 6y = 30$ on the same set of axes.

(c) Graph $x + 4y = 8$, $2x + 8y = 3$, $x - 4y = 6$, and $3x + 12y = 10$ on the same set of axes.

(d) Graph $3x - 4y = 6$, $3x + 4y = 10$, $6x - 8y = 20$, and $6x - 8y = 24$ on the same set of axes.

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(e) For each of the following pairs of lines, (a) predict whether they are parallel lines, and (b) graph each pair of lines to check your prediction.

(1) $5x - 2y = 10$ and $5x - 2y = -4$
(2) $x + y = 6$ and $x - y = 4$
(3) $2x + y = 8$ and $4x + 2y = 2$
(4) $y = 0.2x + 1$ and $y = 0.2x - 4$
(5) $3x - 2y = 4$ and $3x + 2y = 4$
(6) $4x - 3y = 8$ and $8x - 6y = 3$
(7) $2x - y = 10$ and $6x - 3y = 6$
(8) $x + 2y = 6$ and $3x - 6y = 6$

59. Now let's use a graphing calculator to get a graph of $C = \frac{5}{9}(F - 32)$. By letting $F = x$ and $C = y$, we obtain Figure 3.15.

Pay special attention to the boundaries on x. These values were chosen so that the fraction

$$\frac{(\text{Maximum value of } x) \text{ minus } (\text{Minimum value of } x)}{95}$$

would be equal to 1. The viewing window of the graphing calculator used to produce Figure 3.15 is 95 pixels (dots) wide. Therefore, we use 95 as the denominator of the fraction. We chose the boundaries for y to make sure that the cursor would be visible on the screen when we looked for certain values.

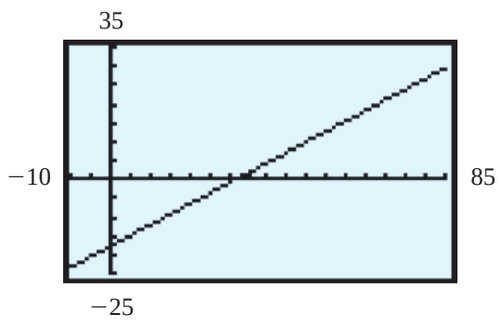


Figure 3.15

Now let's use the TRACE feature of the graphing calculator to complete the following table. Note that the cursor moves in increments of 1 as we trace along the graph.

F	−5	5	9	11	12	20	30	45	60
C									

(This was accomplished by setting the aforementioned fraction equal to 1.) By moving the cursor to each of the F values, we can complete the table as follows.

F	−5	5	9	11	12	20	30	45	60
C	−21	−15	−13	−12	−11	−7	−1	7	16

The C values are expressed to the nearest degree. Use your calculator and check the values in the table by using the equation $C = \frac{5}{9}(F - 32)$.

60. **(a)** Use your graphing calculator to graph $F = \frac{9}{5}C + 32$. Be sure to set boundaries on the horizontal axis so that when you are using the trace feature, the cursor will move in increments of 1.

(b) Use the TRACE feature and check your answers for part (a) of Problem 42.

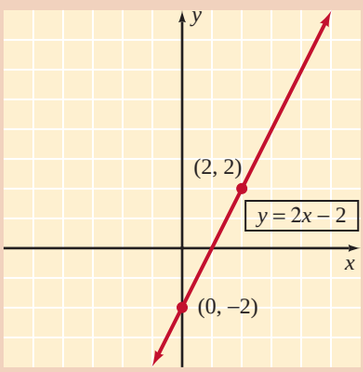
Answers to the Concept Quiz

1. True **2.** False **3.** True **4.** False **5.** True **6.** False **7.** False **8.** False **9.** True **10.** False

Answers to the Example Practice Skills

1. $(-4, 12), (-2, 8), (0, 4), (1, 2), (3, -2)$

2.



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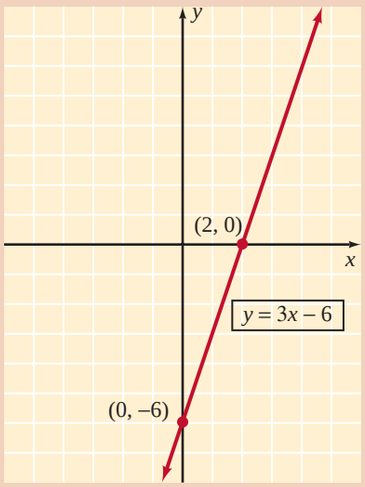
3.



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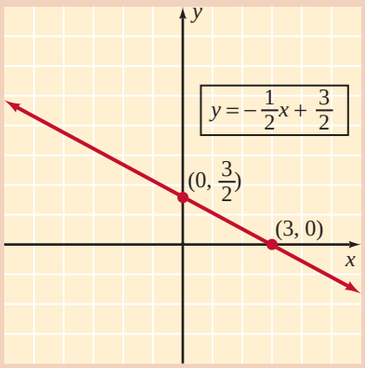

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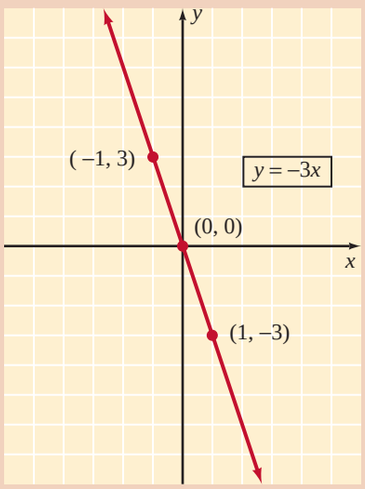

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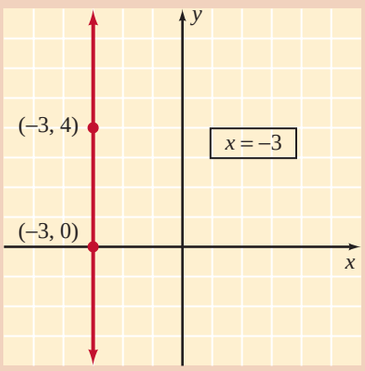

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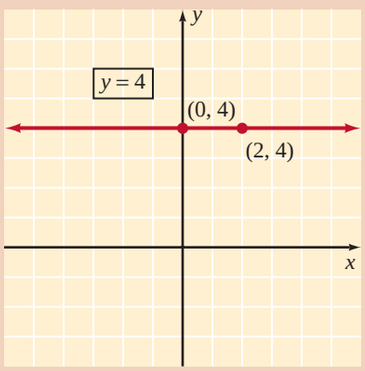
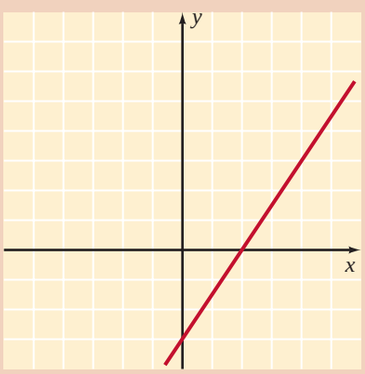


8.



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3.2 Linear Inequalities in Two Variables

OBJECTIVES

1 Graph Linear Inequalities

1 Graph Linear Inequalities

Linear inequalities in two variables are of the form $Ax + By > C$ or $Ax + By < C$, where A, B, and C are real numbers. (Combined linear equality and inequality statements are of the form $Ax + By \geq C$ or $Ax + By \leq C$.)

Graphing linear inequalities is almost as easy as graphing linear equations. The following discussion leads into a simple, step-by-step process. Let's consider the following equation and related inequalities.

$$x + y = 2 \qquad x + y > 2 \qquad x + y < 2$$

The graph of $x + y = 2$ is shown in Figure 3.16. The line divides the plane into two half planes, one above the line and one below the line. In Figure 3.17(a) we have indicated

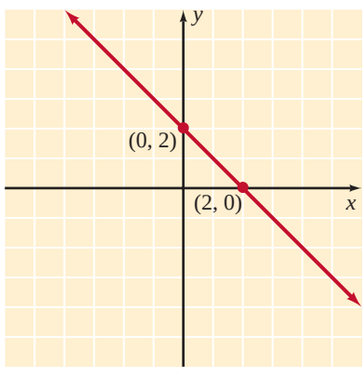
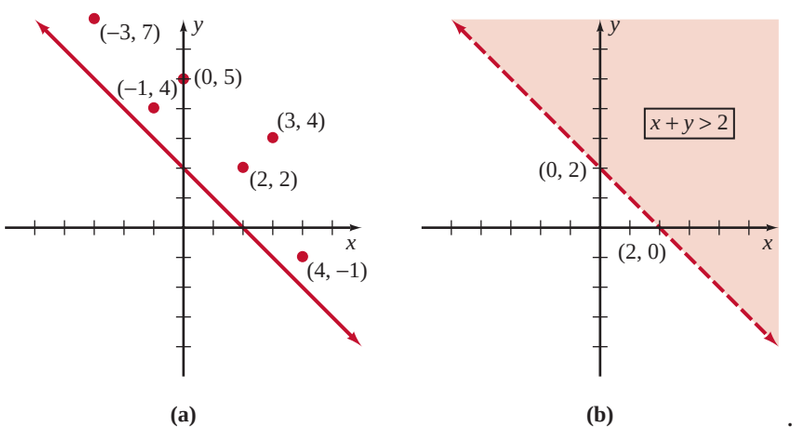


Figure 3.16



(a) **(b)**

Figure 3.17

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several points in the half-plane above the line. Note that for each point, the ordered pair of real numbers satisfies the inequality $x + y > 2$. This is true for *all points* in the half-plane above the line. Therefore, the graph of $x + y > 2$ is the half-plane above the line, as indicated by the shaded portion in Figure 3.17(b). We use a dashed line to indicate that points on the line do *not* satisfy $x + y > 2$. We would use a solid line if we were graphing $x + y \geq 2$.

In Figure 3.18(a), several points were indicated in the half-plane below the line $x + y = 2$. Note that for each point, the ordered pair of real numbers satisfies the inequality $x + y < 2$. This is true for *all points* in the half-plane below the line. Thus the graph of $x + y < 2$ is the half-plane below the line, as indicated in Figure 3.18(b).

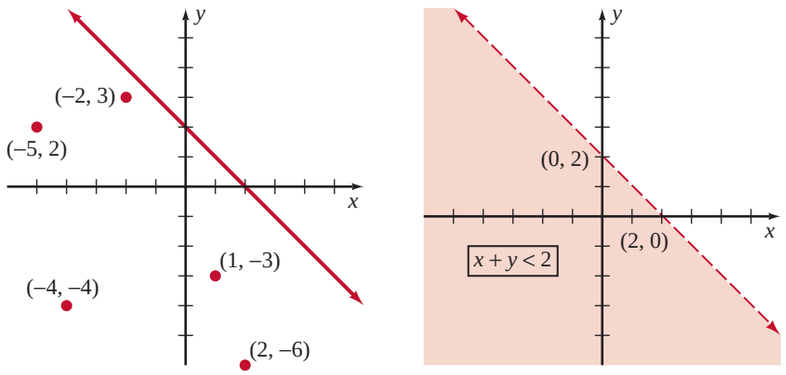

Figure 3.18

To graph a linear inequality, we suggest the following steps.

1. First, graph the corresponding equality. Use a solid line if equality is included in the original statement; use a dashed line if equality is not included.
2. Choose a "test point" not on the line and substitute its coordinates into the inequality. (The origin is a convenient point to use if it is not on the line.)
3. The graph of the original inequality is
   (a) the half-plane that contains the test point if the inequality is satisfied by that point, or
   (b) the half-plane that does not contain the test point if the inequality is not satisfied by the point.

Let's apply these steps to some examples.

EXAMPLE 1

Graph $x - 2y > 4$.

Solution

Step 1 Graph $x - 2y = 4$ as a dashed line because equality is not included in $x - 2y > 4$ (Figure 3.19).

Step 2 Choose the origin as a test point and substitute its coordinates into the inequality.

$x - 2y > 4$ becomes $0 - 2(0) > 4$, which is false.

Step 3 Because the test point did not satisfy the given inequality, the graph is the half-plane that does not contain the test point. Thus the graph of $x - 2y > 4$ is the half-plane below the line, as indicated in Figure 3.19.

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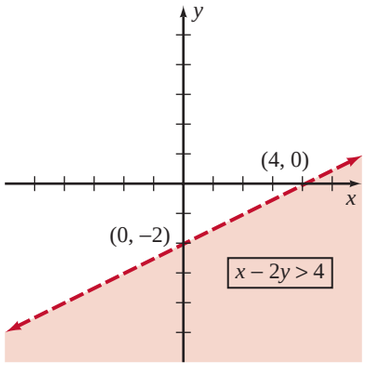

Figure 3.19

▼ PRACTICE YOUR SKILL

Graph $3x + y < 3$. See answer section. ■

EXAMPLE 2

Graph $3x + 2y \leq 6$.

Solution

Step 1 Graph $3x + 2y = 6$ as a solid line because equality is included in $3x + 2y \leq 6$ (Figure 3.20).

Step 2 Choose the origin as a test point and substitute its coordinates into the given statement.

$3x + 2y \leq 6$ becomes $3(0) + 2(0) \leq 6$, which is true.

Step 3 Because the test point satisfies the given statement, all points in the same half-plane as the test point satisfy the statement. Thus the graph of $3x + 2y \leq 6$ consists of the line and the half-plane below the line (Figure 3.20).

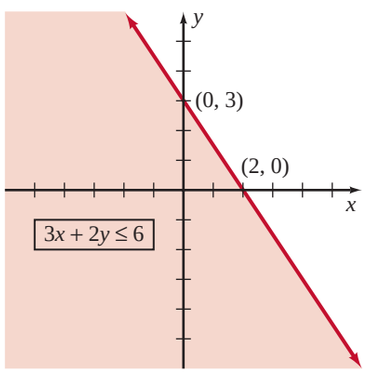

Figure 3.20

▼ PRACTICE YOUR SKILL

Graph $x - 4y \geq 4$. See answer section. ■

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EXAMPLE 3

Graph $y \leq 3x$.

Solution

Step 1 Graph $y = 3x$ as a solid line because equality is included in the statement $y \leq 3x$ (Figure 3.21).

Step 2 The origin is on the line, so we must choose some other point as a test point. Let's try (2, 1).

$y \leq 3x$ becomes $1 \leq 3(2)$, which is a true statement.

Step 3 Because the test point satisfies the given inequality, the graph is the half-plane that contains the test point. Thus the graph of $y \leq 3x$ consists of the line and the half-plane below the line, as indicated in Figure 3.21.

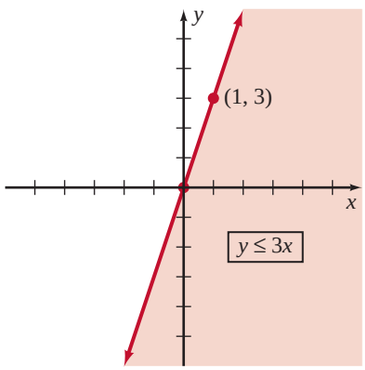

Figure 3.21

▼ PRACTICE YOUR SKILL

Graph $y > 2x$. See answer section. ■

CONCEPT QUIZ

For Problems 1–10, answer true or false.

1. The ordered pair $(2, -3)$ satisfies the inequality $2x + y > 1$.
2. A dashed line on the graph indicates that the points on the line do not satisfy the inequality.
3. Any point can be used as a test point to determine the half-plane that is the solution of the inequality.
4. The ordered pair $(3, -2)$ satisfies the inequality $5x - 2y \geq 19$.
5. The ordered pair $(1, -3)$ satisfies the inequality $-2x - 3y < 4$.
6. The graph of $x > 0$ is the half-plane above the x axis.
7. The graph of $y < 0$ is the half-plane below the x axis.
8. The graph of $-x + y > 4$ is the half-plane above the line $-x + y = 4$.
9. The origin can serve as a test point to determine the half-plane that satisfies the inequality $3y > 2x$.
10. The ordered pair $(-2, -1)$ can be used as a test point to determine the half-plane that satisfies the inequality $y < -3x - 7$.

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Problem Set 3.2

1 Graph Linear Inequalities

For Problems 1–24, graph each of the inequalities.
See answer section.

1. $x - y > 2$

2. $x + y > 4$

3. $x + 3y < 3$

4. $2x - y > 6$

5. $2x + 5y \geq 10$

6. $3x + 2y \leq 4$

7. $y \leq -x + 2$

8. $y \geq -2x - 1$

9. $y > -x$

10. $y < x$

11. $2x - y \geq 0$

12. $x + 2y \geq 0$

13. $-x + 4y - 4 \leq 0$

14. $-2x + y - 3 \leq 0$

15. $y > -\frac{3}{2}x - 3$

16. $2x + 5y > -4$

17. $y < -\frac{1}{2}x + 2$

18. $y < -\frac{1}{3}x + 1$

19. $x \leq 3$

20. $y \geq -2$

21. $x > 1$ and $y < 3$

22. $x > -2$ and $y > -1$

23. $x \leq -1$ and $y < 1$

24. $x < 2$ and $y \geq -2$

THOUGHTS INTO WORDS

25. Why is the point $(-4, 1)$ not a good test point to use when graphing $5x - 2y > -22$?

26. Explain how you would graph the inequality $-3 > x - 3y$.

FURTHER INVESTIGATIONS

27. Graph $|x| < 2$. [*Hint:* Remember that $|x| < 2$ is equivalent to $-2 < x < 2$.] See answer section.

28. Graph $|y| > 1$. See answer section.

29. Graph $|x + y| < 1$. See answer section.

30. Graph $|x - y| > 2$. See answer section.

GRAPHING CALCULATOR ACTIVITIES

31. This is a good time for you to become acquainted with the DRAW features of your graphing calculator. Again, you may need to consult your user's manual for specific key-punching instructions. Return to Examples 1, 2, and 3 of this section, and use your graphing calculator to graph the inequalities.

32. Use a graphing calculator to check your graphs for Problems 1–24.

33. Use the DRAW feature of your graphing calculator to draw each of the following.
(a) A line segment between $(-2, -4)$ and $(-2, 5)$
(b) A line segment between $(2, 2)$ and $(5, 2)$
(c) A line segment between $(2, 3)$ and $(5, 7)$
(d) A triangle with vertices at $(1, -2)$, $(3, 4)$, and $(-3, 6)$

Blue arrows indicate Enhanced WebAssign problems.

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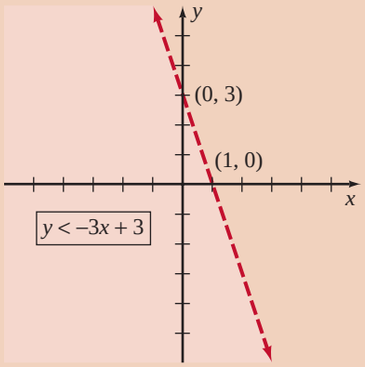
Answers to the Concept Quiz

1. False **2.** True **3.** False **4.** True **5.** False **6.** False **7.** True **8.** True **9.** False **10.** False

Answers to the Example Practice Skills

1.



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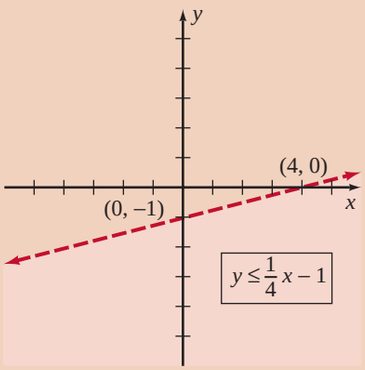

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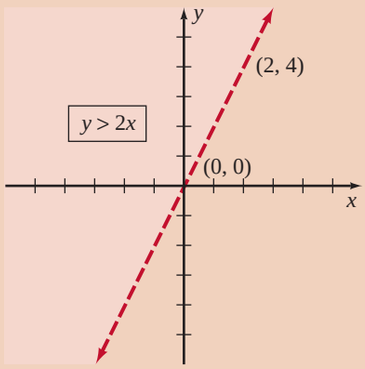

3.3 Distance and Slope

OBJECTIVES

1. Find the Distance between Two Points
2. Find the Slope of a Line
3. Use Slope to Graph Lines
4. Apply Slope to Solve Problems

1 Find the Distance between Two Points

As we work with the rectangular coordinate system, it is sometimes necessary to express the length of certain line segments. In other words, we need to be able to find the distance between two points. Let's first consider two specific examples and then develop the general distance formula.

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EXAMPLE 1

Find the distance between the points $A(2, 2)$ and $B(5, 2)$ and also between the points $C(-2, 5)$ and $D(-2, -4)$.

Solution

Let's plot the points and draw $\overline{AB}$ as in Figure 3.22. Because $\overline{AB}$ is parallel to the x axis, its length can be expressed as $|5 - 2|$ or $|2 - 5|$. (The absolute value is used to ensure a nonnegative value.) Thus the length of $\overline{AB}$ is 3 units. Likewise, the length of $\overline{CD}$ is $|5 - (-4)| = |-4 - 5| = 9$ units.

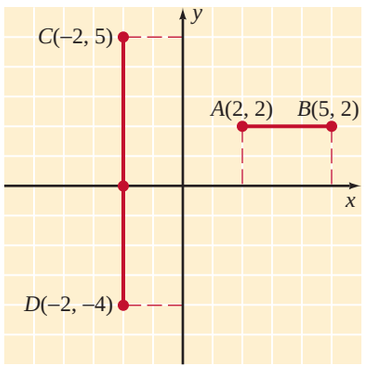


Figure 3.22

▼ PRACTICE YOUR SKILL

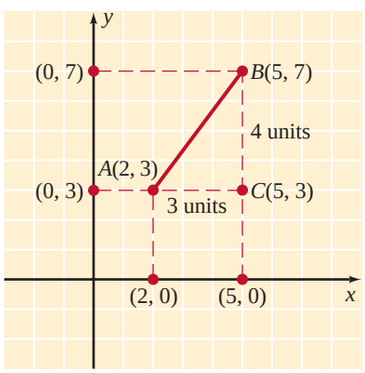
Find the distance between the points $A(3, 6)$ and $B(3, -2)$. 8 units ■

EXAMPLE 2

Find the distance between the points $A(2, 3)$ and $B(5, 7)$.

Solution

Let's plot the points and form a right triangle as indicated in Figure 3.23. Note that the coordinates of point C are $(5, 3)$. Because $\overline{AC}$ is parallel to the horizontal axis, its length is easily determined to be 3 units. Likewise, $\overline{CB}$ is parallel to the vertical axis and its length is 4 units. Let d represent the length of $\overline{AB}$, and apply the Pythagorean theorem to obtain



$$d^2 = 3^2 + 4^2$$

$$d^2 = 9 + 16$$

$$d^2 = 25$$

$$d = \pm\sqrt{25} = \pm 5$$

"Distance between" is a nonnegative value, so the length of $\overline{AB}$ is 5 units.

Figure 3.23

▼ PRACTICE YOUR SKILL

Find the distance between the points $A(-4, 1)$ and $B(8, 6)$. 13 units ■

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We can use the approach we used in Example 2 to develop a general distance formula for finding the distance between any two points in a coordinate plane. The development proceeds as follows:

1. Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ represent any two points in a coordinate plane.
2. Form a right triangle as indicated in Figure 3.24. The coordinates of the vertex of the right angle, point R, are (x_2, y_1).

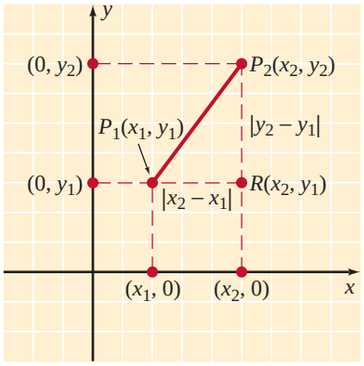

Figure 3.24

The length of $\overline{P_1R}$ is $|x_2 - x_1|$ and the length of $\overline{RP_2}$ is $|y_2 - y_1|$. (Again, the absolute value is used to ensure a nonnegative value.) Let d represent the length of P_1P_2 and apply the Pythagorean theorem to obtain

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Because $|a|^2 = a^2$, the **distance formula** can be stated as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

It makes no difference which point you call P_1 or P_2 when using the distance formula. If you forget the formula, don't panic. Just form a right triangle and apply the Pythagorean theorem as we did in Example 2. Let's consider an example that demonstrates the use of the distance formula.

Answers to the distance problems can be left in square-root form or approximated using a calculator. Radical answers in this chapter will be restricted to radicals that are perfect squares or radicals that do not need to be simplified. The skill of simplifying radicals is covered in Chapter 7, after which you will be able to simplify the answers for distance problems.

EXAMPLE 3

Find the distance between $(-1, 5)$ and $(1, 2)$.

Solution

Let $(-1, 5)$ be P_1 and $(1, 2)$ be P_2. Using the distance formula, we obtain

$$\begin{aligned} d &= \sqrt{[(1 - (-1))]^2 + (2 - 5)^2} \\ &= \sqrt{2^2 + (-3)^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \end{aligned}$$

The distance between the two points is $\sqrt{13}$ units.

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▼ PRACTICE YOUR SKILL

Find the distance between the points $A(-1, 3)$ and $B(4, 5)$. $\sqrt{29}$ units ■

In Example 3, we did not sketch a figure because of the simplicity of the problem. However, sometimes it is helpful to use a figure to organize the given information and aid in analyzing the problem, as we see in the next example.

EXAMPLE 4

Verify that the points (2, 2), (11, 7), and (4, 9) are vertices of an isosceles triangle. (An isosceles triangle has two sides of the same length.)

Solution

Let's plot the points and draw the triangle (Figure 3.25). Use the distance formula to find the lengths d_1, d_2, and d_3, as follows:

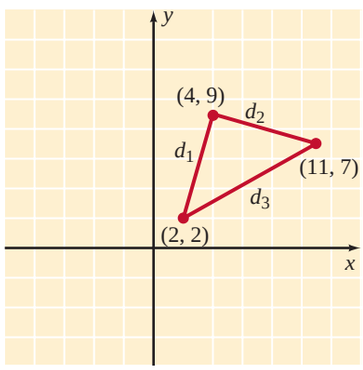

Figure 3.25

$$\begin{aligned} d_1 &= \sqrt{(4-2)^2 + (9-2)^2} \\ &= \sqrt{2^2 + 7^2} \\ &= \sqrt{4 + 49} = \sqrt{53} \\ d_2 &= \sqrt{(11-4)^2 + (7-9)^2} \\ &= \sqrt{7^2 + (-2)^2} \\ &= \sqrt{49 + 4} = \sqrt{53} \\ d_3 &= \sqrt{(11-2)^2 + (7-2)^2} \\ &= \sqrt{9^2 + 5^2} \\ &= \sqrt{81 + 25} = \sqrt{106} \end{aligned}$$

Because $d_1 = d_2$, we know that it is an isosceles triangle.

▼ PRACTICE YOUR SKILL

Verify that the points $(-2, -2)$, $(7, -1)$, and $(2, 3)$ are vertices of an isosceles triangle. $d_1 = d_2 = \sqrt{41}$, $d_3 = \sqrt{82}$ ■

2 Find the Slope of a Line

In coordinate geometry, the concept of **slope** is used to describe the "steepness" of lines. The slope of a line is the ratio of the vertical change to the horizontal change as we move from one point on a line to another point. This is illustrated in Figure 3.26 with points P_1 and P_2.

A precise definition for slope can be given by considering the coordinates of the points P_1, P_2, and R as indicated in Figure 3.27. The horizontal change as we move from P_1 to P_2 is $x_2 - x_1$ and the vertical change is $y_2 - y_1$. Thus the following definition for slope is given.

Definition 3.1

If points P_1 and P_2 with coordinates (x_1, y_1) and (x_2, y_2), respectively, are any two different points on a line, then the slope of the line (denoted by m) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_2 \neq x_1$$

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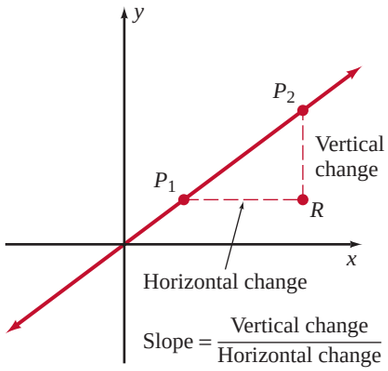

Figure 3.26

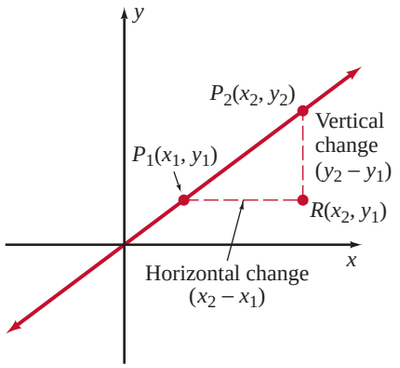

Figure 3.27

Because $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$, how we designate P_1 and P_2 is not important. Let's use Definition 3.1 to find the slopes of some lines.

EXAMPLE 5

Find the slope of the line determined by each of the following pairs of points, and graph the lines.

(a) $(-1, 1)$ and $(3, 2)$ **(b)** $(4, -2)$ and $(-1, 5)$

(c) $(2, -3)$ and $(-3, -3)$

Solution

(a) Let $(-1, 1)$ be P_1 and $(3, 2)$ be P_2 (Figure 3.28).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{3 - (-1)} = \frac{1}{4}$$

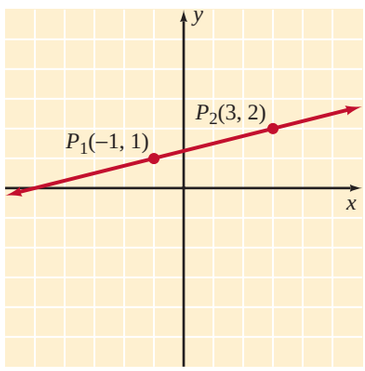

Figure 3.28

(b) Let $(4, -2)$ be P_1 and $(-1, 5)$ be P_2 (Figure 3.29).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{-1 - 4} = \frac{7}{-5} = -\frac{7}{5}$$

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(c) Let $(2, -3)$ be P_1 and $(-3, -3)$ be P_2 (Figure 3.30).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-3)}{-3 - 2} = \frac{0}{-5} = 0$$

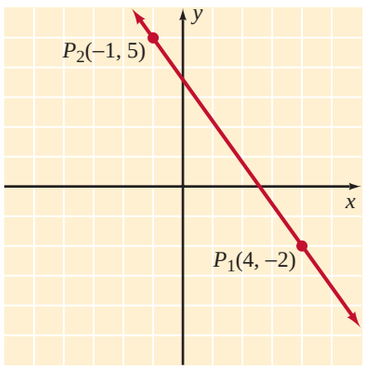

Figure 3.29

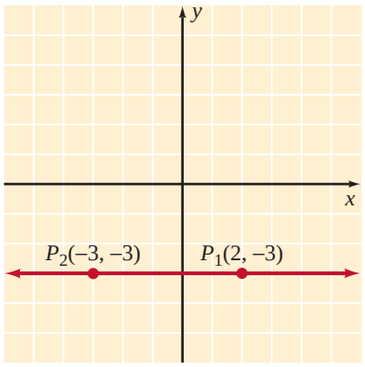

Figure 3.30

▼ PRACTICE YOUR SKILL

Find the slope of the line determined by each of the following pairs of points, and graph the lines.

(a) $(-4, 2)$ and $(2, 5)$ **(b)** $(-3, 4)$ and $(1, 4)$
(c) $(3, -2)$ and $(0, 2)$ See answer section. ■

The three parts of Example 5 represent the three basic possibilities for slope; that is, the slope of a line can be positive, negative, or zero. A line that has a positive slope rises as we move from left to right, as in Figure 3.28. A line that has a negative slope falls as we move from left to right, as in Figure 3.29. A horizontal line, as in Figure 3.30, has a slope of zero. Finally, we need to realize that *the concept of slope is undefined for vertical lines*. This is due to the fact that for any vertical line, the horizontal change as we move from one point on the line to another is zero. Thus the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ will have a denominator of zero and be undefined. Accordingly, the restriction $x_2 \neq x_1$ is imposed in Definition 3.1.

One final idea pertaining to the concept of slope needs to be emphasized. The slope of a line is a **ratio**, the ratio of vertical change to horizontal change. A slope of $\frac{2}{3}$ means that for every 2 units of vertical change there must be a corresponding 3 units of horizontal change. Thus, starting at some point on a line that has a slope of $\frac{2}{3}$, we could locate other points on the line as follows:

$\frac{2}{3} = \frac{4}{6}$ → by moving 4 units *up* and 6 units to the *right*
$\frac{2}{3} = \frac{8}{12}$ → by moving 8 units *up* and 12 units to the *right*
$\frac{2}{3} = \frac{-2}{-3}$ → by moving 2 units *down* and 3 units to the *left*

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Likewise, if a line has a slope of $-\frac{3}{4}$, then by starting at some point on the line we could locate other points on the line as follows:

$-\frac{3}{4} = \frac{-3}{4}$ → by moving 3 units *down* and 4 units to the *right*

$-\frac{3}{4} = \frac{3}{-4}$ → by moving 3 units *up* and 4 units to the *left*

$-\frac{3}{4} = \frac{-9}{12}$ → by moving 9 units *down* and 12 units to the *right*

$-\frac{3}{4} = \frac{15}{-20}$ → by moving 15 units *up* and 20 units to the *left*

3 Use Slope to Graph Lines

EXAMPLE 6 Graph the line that passes through the point $(0, -2)$ and has a slope of $\frac{1}{3}$.

Solution

To graph, plot the point $(0, -2)$. Furthermore, because the slope is equal to $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{1}{3}$, we can locate another point on the line by starting from the point $(0, -2)$ and moving 1 unit up and 3 units to the right to obtain the point $(3, -1)$. Because two points determine a line, we can draw the line (Figure 3.31).

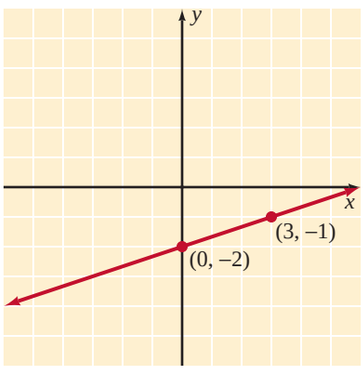

Figure 3.31

Remark: Because $m = \frac{1}{3} = \frac{-1}{-3}$, we can locate another point by moving 1 unit down and 3 units to the left from the point $(0, -2)$.

▼ PRACTICE YOUR SKILL

Graph the line that passes through the point $(-3, 2)$ and has a slope of $\frac{2}{5}$. ■

See answer section.

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EXAMPLE 7 Graph the line that passes through the point (1, 3) and has a slope of −2.

Solution

To graph the line, plot the point (1, 3). We know that $m = -2 = \frac{-2}{1}$. Furthermore, because the slope $= \frac{\text{vertical change}}{\text{horizontal change}} = \frac{-2}{1}$, we can locate another point on the line by starting from the point (1, 3) and moving 2 units down and 1 unit to the right to obtain the point (2, 1). Because two points determine a line, we can draw the line (Figure 3.32).

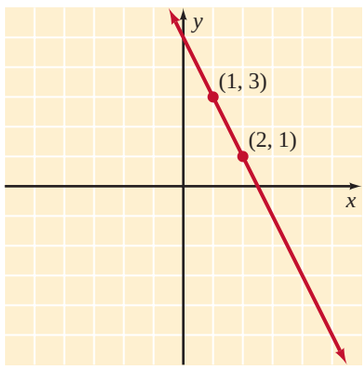

Figure 3.32

Remark: Because $m = -2 = \frac{-2}{1} = \frac{2}{-1}$ we can locate another point by moving 2 units up and 1 unit to the left from the point (1, 3).

▼ **PRACTICE YOUR SKILL**

Graph the line that passes through the point (2, 0) and has a slope of $m = -\frac{1}{3}$. ■

See answer section.

4 Apply Slope to Solve Problems

The concept of slope has many real-world applications even though the word *slope* is often not used. The concept of slope is used in most situations where an incline is involved. Hospital beds are hinged in the middle so that both the head end and the foot end can be raised or lowered; that is, the slope of either end of the bed can be changed. Likewise, treadmills are designed so that the incline (slope) of the platform can be adjusted. A roofer, when making an estimate to replace a roof, is concerned not only about the total area to be covered but also about the pitch of the roof. (Contractors do not define *pitch* as identical with the mathematical definition of slope, but both concepts refer to "steepness.") In Figure 3.33, the two roofs might require the same amount of shingles, but the roof on the left will take longer to complete because the pitch is so great that scaffolding will be required.

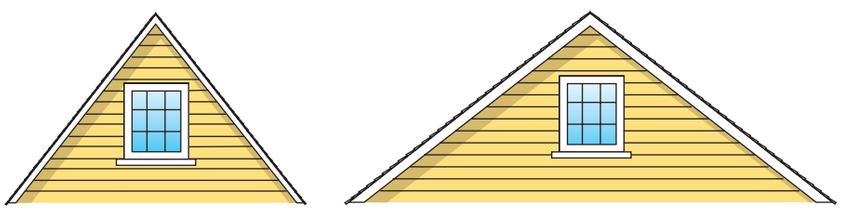

Figure 3.33

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The concept of slope is also used in the construction of flights of stairs (Figure 3.34). The terms *rise* and *run* are commonly used, and the steepness (slope) of the stairs can be expressed as the ratio of rise to run. In Figure 3.34, the stairs on the left, where the ratio of rise to run is $\frac{10}{11}$, are steeper than the stairs on the right, which have a ratio of $\frac{7}{11}$.

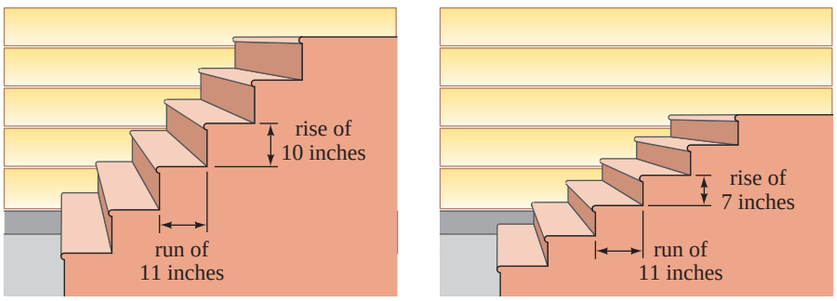

Figure 3.34

In highway construction, the word *grade* is used for the concept of slope. For example, in Figure 3.35 the highway is said to have a grade of 17%. This means that for every horizontal distance of 100 feet, the highway rises or drops 17 feet. In other words, the slope of the highway is $\frac{17}{100}$.

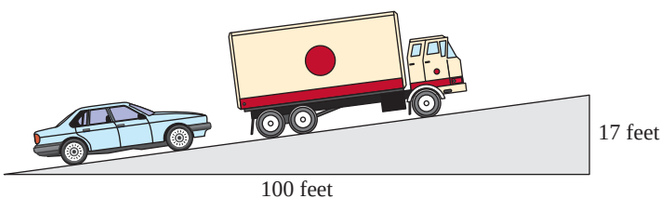

Figure 3.35

EXAMPLE 8 A certain highway has a 3% grade. How many feet does it rise in a horizontal distance of 1 mile?

Solution

A 3% grade means a slope of $\frac{3}{100}$. Therefore, if we let y represent the unknown vertical distance and use the fact that 1 mile = 5280 feet, we can set up and solve the following proportion.

$$\frac{3}{100} = \frac{y}{5280}$$
$$100y = 3(5280) = 15{,}840$$
$$y = 158.4$$

The highway rises 158.4 feet in a horizontal distance of 1 mile.

▼ PRACTICE YOUR SKILL

A certain highway has a 2.5% grade. How many feet does it rise in a horizontal distance of 2000 feet? 50 ft ■

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CONCEPT QUIZ

For Problems 1–10, answer true or false.

1. When applying the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the distance between two points, you can designate either of the two points as P_1.
2. An isosceles triangle has two sides of the same length.
3. The distance between the points $(-1, 4)$ and $(-1, -2)$ is 2 units.
4. The distance between the points $(3, -4)$ and $(3, 2)$ is undefined.
5. The slope of a line is the ratio of the vertical change to the horizontal change when moving from one point on the line to another point on the line.
6. The slope of a line is always positive.
7. A slope of 0 means that there is no change in the vertical direction when moving from one point on the line to another point on the line.
8. The concept of slope is undefined for horizontal lines.
9. When applying the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the slope of a line between two points, you can designate either of the two points as P_2.
10. If the ratio of the rise to the run for some steps is $\dfrac{3}{4}$ and the rise is 9 inches, then the run is $6\dfrac{3}{4}$ inches.

Problem Set 3.3

1 Find the Distance between Two Points

For Problems 1–12, find the distance between each of the pairs of points. Express answers in radical form.

▶ **1.** $(-2, -1), (7, 11)$ 15

2. $(2, 1), (10, 7)$ 10

3. $(1, -1), (3, -4)$ $\sqrt{13}$

4. $(-1, 3), (2, -2)$ $\sqrt{34}$

▶ **5.** $(6, -5), (9, -7)$ $\sqrt{97}$

6. $(-4, 2), (-1, 6)$ 5

7. $(-3, 3), (0, -2)$ $\sqrt{34}$

8. $(-1, -4), (4, 0)$ $\sqrt{41}$

9. $(1, -6), (-5, -6)$ 6

10. $(-2, 3), (-2, -7)$ 10

11. $(1, 7), (4, -1)$ $\sqrt{73}$

▶ **12.** $(6, 4), (3, -8)$ $\sqrt{153}$

▶ **13.** Verify that the points $(0, 2)$, $(0, 7)$, and $(12, 7)$ are vertices of a right triangle. [*Hint:* If $a^2 + b^2 = c^2$, then it is a right triangle with the right angle opposite side c.]

14. Verify that the points $(0, 4)$, $(-3, 0)$, and $(3, 0)$ are vertices of an isosceles triangle.

15. Verify that the points $(3, 5)$ and $(5, 8)$ divide the line segment joining $(1, 2)$ and $(7, 11)$ into three segments of equal length.

16. Verify that $(5, 1)$ is the midpoint of the line segment joining $(2, 6)$ and $(8, -4)$.

2 Find the Slope of a Line

For Problems 17–28, graph the line determined by the two points and find the slope of the line.

▶ **17.** $(1, 2), (4, 6)$ $\frac{4}{3}$

18. $(3, 1), (-2, -2)$ $\frac{3}{5}$

19. $(-4, 5), (-1, -2)$ $-\frac{7}{3}$

20. $(-2, 5), (3, -1)$ $-\frac{6}{5}$

▶ **21.** $(2, 6), (6, -2)$ -2

22. $(-2, -1), (2, -5)$ -1

23. $(-6, 1), (-1, 4)$ $\frac{3}{5}$

24. $(-3, 3), (2, 3)$ 0

25. $(-2, -4), (2, -4)$ 0

26. $(1, -5), (4, -1)$ $\frac{4}{3}$

27. $(0, -2), (4, 0)$ $\frac{1}{2}$

▶ **28.** $(-4, 0), (0, -6)$ $-\frac{3}{2}$

▶ **29.** Find x if the line through $(-2, 4)$ and $(x, 6)$ has a slope of $\dfrac{2}{9}$. 7

30. Find y if the line through $(1, y)$ and $(4, 2)$ has a slope of $\dfrac{5}{3}$. -3

31. Find x if the line through $(x, 4)$ and $(2, -5)$ has a slope of $-\dfrac{9}{4}$. -2

▶ **32.** Find y if the line through $(5, 2)$ and $(-3, y)$ has a slope of $-\dfrac{7}{8}$. 9

For Problems 33 – 40, you are given one point on a line and the slope of the line. Find the coordinates of three other points on the line. Answers vary

33. $(2, 5)$, $m = \dfrac{1}{2}$

34. $(3, 4)$, $m = \dfrac{5}{6}$

35. $(-3, 4)$, $m = 3$

36. $(-3, -6)$, $m = 1$

37. $(5, -2)$, $m = -\dfrac{2}{3}$

38. $(4, -1)$, $m = -\dfrac{3}{4}$

39. $(-2, -4)$, $m = -2$

40. $(-5, 3)$, $m = -3$

▶Blue arrows indicate Enhanced WebAssign problems.

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For Problems 41–50, find the coordinates of two points on the given line, and then use those coordinates to find the slope of the line.

41. $2x + 3y = 6$ $-\frac{2}{3}$

42. $4x + 5y = 20$ $-\frac{4}{5}$

43. $x - 2y = 4$ $\frac{1}{2}$

44. $3x - y = 12$ 3

45. $4x - 7y = 12$ $\frac{4}{7}$

46. $2x + 7y = 11$ $-\frac{2}{7}$

47. $y = 4$ 0

48. $x = 3$ Undefined

49. $y = -5x$ -5

50. $y - 6x = 0$ 6

3 Use Slope to Graph Lines

For Problems 51–58, graph the line that passes through the given point and has the given slope. See answer section.

51. $(3, 1)$ $m = \frac{2}{3}$

52. $(-1, 0)$ $m = \frac{3}{4}$

53. $(-2, 3)$ $m = -1$

54. $(1, -4)$ $m = -3$

55. $(0, 5)$ $m = \frac{-1}{4}$

56. $(-3, 4)$ $m = \frac{-3}{2}$

57. $(2, -2)$ $m = \frac{3}{2}$

58. $(3, -4)$ $m = \frac{5}{2}$

4 Apply Slope to Solve Problems

59. A certain highway has a 2% grade. How many feet does it rise in a horizontal distance of 1 mile? (1 mile = 5280 feet) 105.6 ft

60. The grade of a highway up a hill is 30%. How much change in horizontal distance is there if the vertical height of the hill is 75 feet? 250 ft

61. Suppose that a highway rises a distance of 215 feet in a horizontal distance of 2640 feet. Express the grade of the highway to the nearest tenth of a percent. 8.1%

62. If the ratio of rise to run is to be $\frac{3}{5}$ for some steps and the rise is 19 centimeters, find the run to the nearest centimeter. 32 cm

63. If the ratio of rise to run is to be $\frac{2}{3}$ for some steps and the run is 28 centimeters, find the rise to the nearest centimeter. 19 cm

64. Suppose that a county ordinance requires a $2\frac{1}{4}\%$ "fall" for a sewage pipe from the house to the main pipe at the street. How much vertical drop must there be for a horizontal distance of 45 feet? Express the answer to the nearest tenth of a foot. 1.0 ft

THOUGHTS INTO WORDS

65. How would you explain the concept of slope to someone who was absent from class the day it was discussed?

66. If one line has a slope of $\frac{2}{5}$ and another line has a slope of $\frac{3}{7}$, which line is steeper? Explain your answer.

67. Suppose that a line has a slope of $\frac{2}{3}$ and contains the point (4, 7). Are the points (7, 9) and (1, 3) also on the line? Explain your answer.

FURTHER INVESTIGATIONS

68. Sometimes it is necessary to find the coordinate of a point on a number line that is located somewhere between two given points. For example, suppose that we want to find the coordinate (x) of the point located two-thirds of the distance from 2 to 8. Because the total distance from 2 to 8 is $8 - 2 = 6$ units, we can start at 2 and move $\frac{2}{3}(6) = 4$ units toward 8. Thus $x = 2 + \frac{2}{3}(6) = 2 + 4 = 6$.

For each of the following, find the coordinate of the indicated point on a number line.

(a) Two-thirds of the distance from 1 to 10 7

(b) Three-fourths of the distance from -2 to 14 10

(c) One-third of the distance from -3 to 7 $\frac{1}{3}$

(d) Two-fifths of the distance from -5 to 6 $-\frac{3}{5}$

(e) Three-fifths of the distance from -1 to -11 -7

(f) Five-sixths of the distance from 3 to -7 $-\frac{16}{3}$

69. Now suppose that we want to find the coordinates of point P, which is located two-thirds of the distance from $A(1, 2)$ to $B(7, 5)$ in a coordinate plane. We have plotted the given points A and B in Figure 3.36 to help with the analysis of this problem. Point D is two-thirds of the distance from A to C because parallel lines cut off proportional segments on every transversal that intersects the lines. Thus $\overline{AC}$ can be treated as a segment of a number line, as shown in Figure 3.37.

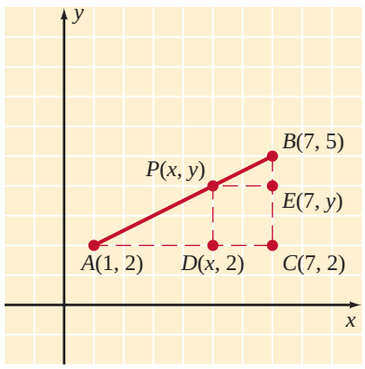

Figure 3.36

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Figure 3.37

Therefore,

$$x = 1 + \frac{2}{3}(7 - 1) = 1 + \frac{2}{3}(6) = 5$$

Similarly, $\overline{CB}$ can be treated as a segment of a number line, as shown in Figure 3.38. Therefore,

$$y = 2 + \frac{2}{3}(5 - 2) = 2 + \frac{2}{3}(3) = 4$$

The coordinates of point P are $(5, 4)$.

Figure 3.38

For each of the following, find the coordinates of the indicated point in the xy plane.

(a) One-third of the distance from $(2, 3)$ to $(5, 9)$ (3, 5)
(b) Two-thirds of the distance from $(1, 4)$ to $(7, 13)$ (5, 10)
(c) Two-fifths of the distance from $(-2, 1)$ to $(8, 11)$ (2, 5)
(d) Three-fifths of the distance from $(2, -3)$ to $(-3, 8)$ See below
(e) Five-eighths of the distance from $(-1, -2)$ to $(4, -10)$ See below
(f) Seven-eighths of the distance from $(-2, 3)$ to $(-1, -9)$ See below

70. Suppose we want to find the coordinates of the midpoint of a line segment. Let $P(x, y)$ represent the midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$. Using the method in Problem 68, the formula for the x coordinate of the midpoint is $x = x_1 + \frac{1}{2}(x_2 - x_1)$. This formula can be manipulated algebraically to produce a simpler formula:

$$x = x_1 + \frac{1}{2}(x_2 - x_1)$$

$$x = x_1 + \frac{1}{2}x_2 - \frac{1}{2}x_1$$

$$x = \frac{1}{2}x_1 + \frac{1}{2}x_2$$

$$x = \frac{x_1 + x_2}{2}$$

Hence the x coordinate of the midpoint can be interpreted as the average of the x coordinates of the endpoints of the line segment. A similar argument for the y coordinate of the midpoint gives the following formula:

$$y = \frac{y_1 + y_2}{2}$$

For each of the pairs of points, use the formula to find the midpoint of the line segment between the points.

(a) $(3, 1)$ and $(7, 5)$ (5, 3)
(b) $(-2, 8)$ and $(6, 4)$ (2, 6)
(c) $(-3, 2)$ and $(5, 8)$ (1, 5)
(d) $(4, 10)$ and $(9, 25)$ $\left(\frac{13}{2}, \frac{35}{2}\right)$
(e) $(-4, -1)$ and $(-10, 5)$ (−7, 2)
(f) $(5, 8)$ and $(-1, 7)$ $\left(2, \frac{15}{2}\right)$

GRAPHING CALCULATOR ACTIVITIES

71. Remember that we did some work with parallel lines back in the graphing calculator activities in Problem Set 3.1. Now let's do some work with perpendicular lines. Be sure to set your boundaries so that the distance between tick marks is the same on both axes.

(a) Graph $y = 4x$ and $y = -\frac{1}{4}x$ on the same set of axes. Do they appear to be perpendicular lines?

(b) Graph $y = 3x$ and $y = \frac{1}{3}x$ on the same set of axes. Do they appear to be perpendicular lines?

(c) Graph $y = \frac{2}{5}x - 1$ and $y = -\frac{5}{2}x + 2$ on the same set of axes. Do they appear to be perpendicular lines?

(d) Graph $y = \frac{3}{4}x - 3$, $y = \frac{4}{3}x + 2$, and $y = -\frac{4}{3}x + 2$ on the same set of axes. Does there appear to be a pair of perpendicular lines?

(e) On the basis of your results in parts (a) through (d), make a statement about how we can recognize perpendicular lines from their equations.

▶72. For each of the following pairs of equations: (1) predict whether they represent parallel lines, perpendicular lines, or lines that intersect but are not perpendicular; and (2) graph each pair of lines to check your prediction.

(a) $5.2x + 3.3y = 9.4$ and $5.2x + 3.3y = 12.6$
(b) $1.3x - 4.7y = 3.4$ and $1.3x - 4.7y = 11.6$
(c) $2.7x + 3.9y = 1.4$ and $2.7x - 3.9y = 8.2$
(d) $5x - 7y = 17$ and $7x + 5y = 19$
(e) $9x + 2y = 14$ and $2x + 9y = 17$
(f) $2.1x + 3.4y = 11.7$ and $3.4x - 2.1y = 17.3$

69. **(d)** $\left(-1, \frac{18}{5}\right)$ **(e)** $\left(\frac{17}{8}, -7\right)$ **(f)** $\left(-\frac{9}{8}, -\frac{15}{2}\right)$

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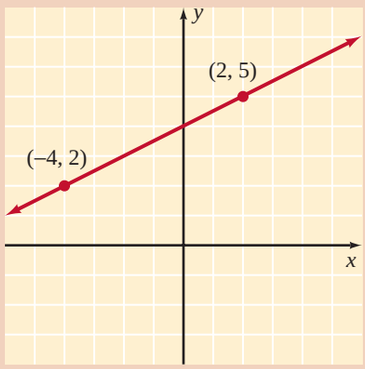
Answers to the Concept Quiz

1. True **2.** True **3.** False **4.** False **5.** True **6.** False **7.** True **8.** False **9.** True **10.** False

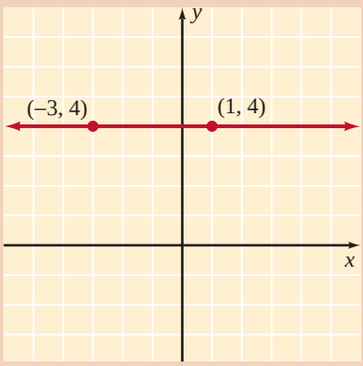
Answers to the Example Practice Skills

1. 8 units **2.** 13 units **3.** $\sqrt{29}$ units **4.** $d_1 = d_2 = \sqrt{41}, d_3 = \sqrt{82}$

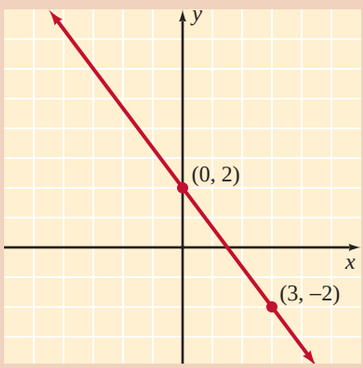
5. (a) $m = \frac{1}{2}$



(b) $m = 0$



(c) $m = -\frac{4}{3}$



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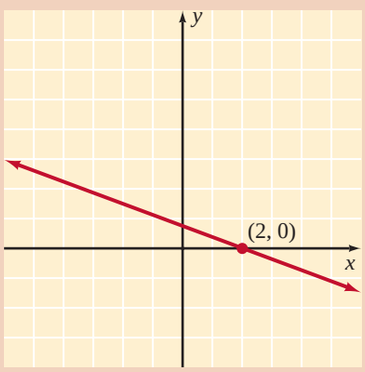

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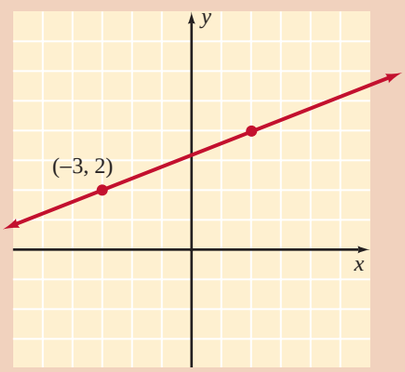

8. 50 ft

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3.4 Determining the Equation of a Line

OBJECTIVES

1. Find the Equation of a Line Given a Point and a Slope
2. Find the Equation of a Line Given Two Points
3. Find the Equation of a Line Given the Slope and *y* Intercept
4. Use the Point-Slope Form to Write Equations of Lines
5. Apply the Slope-Intercept Form of an Equation
6. Find the Equations for Parallel or Perpendicular Lines

1 Find the Equation of a Line Given a Point and a Slope

To review, there are basically two types of problems to solve in coordinate geometry:

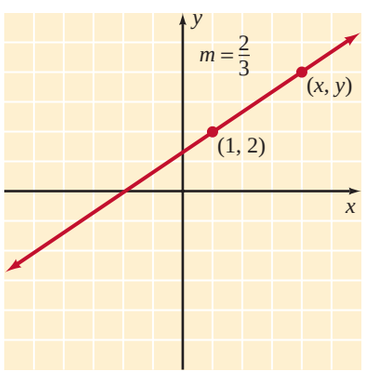
1. Given an algebraic equation, find its geometric graph.
2. Given a set of conditions pertaining to a geometric figure, find its algebraic equation.

Problems of type 1 have been our primary concern thus far in this chapter. Now let's analyze some problems of type 2 that deal specifically with straight lines. Given certain facts about a line, we need to be able to determine its algebraic equation. Let's consider some examples.

EXAMPLE 1 Find the equation of the line that has a slope of $\frac{2}{3}$ and contains the point $(1, 2)$.

Solution

First, let's draw the line and record the given information. Then choose a point (x, y) that represents any point on the line other than the given point $(1, 2)$. (See Figure 3.39.)



The slope determined by $(1, 2)$ and (x, y) is $\frac{2}{3}$. Thus

$$\frac{y-2}{x-1} = \frac{2}{3}$$

$$2(x-1) = 3(y-2)$$

$$2x - 2 = 3y - 6$$

$$2x - 3y = -4$$

Figure 3.39

▼ **PRACTICE YOUR SKILL**

Find the equation of the line that has a slope of $\frac{3}{4}$ and contains the point $(3, -1)$. ■

$3x - 4y = 13$

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2 Find the Equation of a Line Given Two Points

EXAMPLE 2 Find the equation of the line that contains (3, 2) and (−2, 5).

Solution

First, let's draw the line determined by the given points (Figure 3.40); if we know two points, we can find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{-5} = -\frac{3}{5}$$

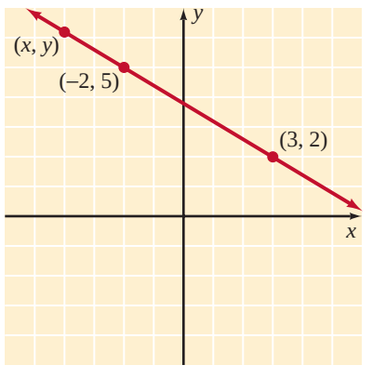

Figure 3.40

Now we can use the same approach as in Example 1.

Form an equation using a variable point (x, y), one of the two given points, and the slope of $-\frac{3}{5}$.

$$\frac{y - 5}{x + 2} = \frac{3}{-5} \qquad \left(-\frac{3}{5} = \frac{3}{-5}\right)$$

$$3(x + 2) = -5(y - 5)$$

$$3x + 6 = -5y + 25$$

$$3x + 5y = 19$$

▼ **PRACTICE YOUR SKILL**

Find the equation of the line that contains (2, 5) and (−4, 10). $5x + 6y = 40$ ■

3 Find the Equation of a Line Given the Slope and *y* Intercept

EXAMPLE 3 Find the equation of the line that has a slope of $\frac{1}{4}$ and a y intercept of 2.

Solution

A y intercept of 2 means that the point (0, 2) is on the line (Figure 3.41).

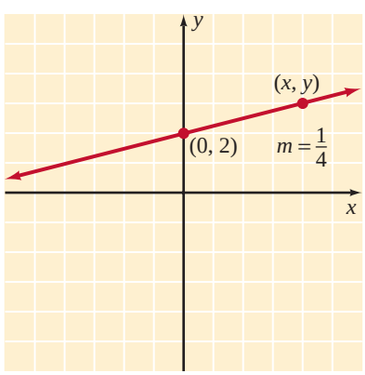

Figure 3.41

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Choose a variable point (x, y) and proceed as in the previous examples.

$$\frac{y-2}{x-0} = \frac{1}{4}$$
$$1(x-0) = 4(y-2)$$
$$x = 4y - 8$$
$$x - 4y = -8$$

▼ PRACTICE YOUR SKILL

Find the equation of the line that has a slope of $\frac{3}{2}$ and a y intercept of -4. ■

$3x - 2y = 8$

Perhaps it would be helpful to pause a moment and look back over Examples 1, 2, and 3. Note that we used the same basic approach in all three situations. We chose a variable point (x, y) and used it to determine the equation that satisfies the conditions given in the problem. The approach we took in the previous examples can be generalized to produce some special forms of equations of straight lines.

4 Use the Point-Slope Form to Write Equations of Lines

Generalizing from the previous examples, let's find the equation of a line that has a slope of m and contains the point (x_1, y_1). To use the slope formula we will need two points. Choosing a point (x, y) to represent any other point on the line (Figure 3.42) and using the given point (x_1, y_1), we can determine slope to be

$$m = \frac{y - y_1}{x - x_1}, \quad \text{where } x \neq x_1$$

Simplifying gives us the equation $y - y_1 = m(x - x_1)$.

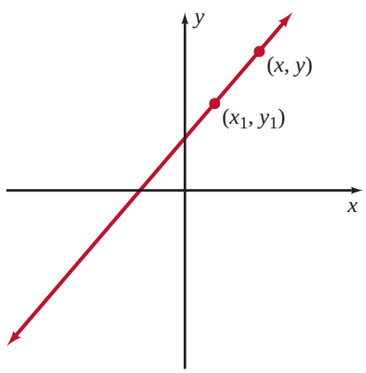

Figure 3.42

We refer to the equation

$$y - y_1 = m(x - x_1)$$

as the **point-slope form** of the equation of a straight line. Instead of the approach we used in Example 1, we could use the point-slope form to write the equation of a line with a given slope that contains a given point.

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EXAMPLE 4

Use the point-slope form to find the equation of a line that has a slope of $\frac{3}{5}$ and contains the point (2, 4).

Solution

We can determine the equation of the line by substituting $\frac{3}{5}$ for m and (2, 4) for (x_1, y_1) in the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= \frac{3}{5}(x - 2) \\ 5(y - 4) &= 3(x - 2) \\ 5y - 20 &= 3x - 6 \\ -14 &= 3x - 5y \end{aligned}$$

Thus the equation of the line is $3x - 5y = -14$.

▼ **PRACTICE YOUR SKILL**

Use the point-slope form to find the equation of a line that has a slope of $\frac{4}{3}$ and contains the point $(-2, 5)$. $4x - 3y = -23$ ■

5 Apply the Slope-Intercept Form of an Equation

Another special form of the equation of a line is the slope-intercept form. Let's use the point-slope form to find the equation of a line that has a slope of m and a y intercept of b. A y intercept of b means that the line contains the point $(0, b)$, as in Figure 3.43. Therefore, we can use the point-slope form as follows:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - b &= m(x - 0) \\ y - b &= mx \\ y &= mx + b \end{aligned}$$

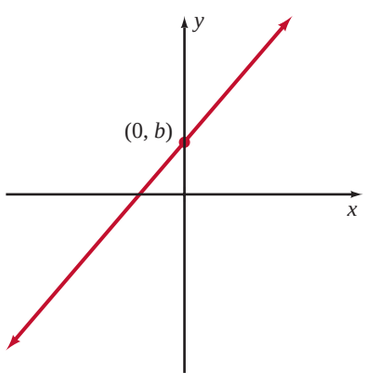

Figure 3.43

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We refer to the equation

$$y = mx + b$$

as the **slope-intercept form** of the equation of a straight line. We use it for three primary purposes, as the next three examples illustrate.

EXAMPLE 5

Find the equation of the line that has a slope of $\frac{1}{4}$ and a y intercept of 2.

Solution

This is a restatement of Example 3, but this time we will use the slope-intercept form ($y = mx + b$) of a line to write its equation. Because $m = \frac{1}{4}$ and $b = 2$, we can substitute these values into $y = mx + b$.

$$y = mx + b$$

$$y = \frac{1}{4}x + 2$$

$$4y = x + 8 \qquad \text{Multiply both sides by 4}$$

$$x - 4y = -8 \qquad \text{Same result as in Example 3}$$

▼ PRACTICE YOUR SKILL

Find the equation of the line that has a slope of -3 and a y intercept of 8. ■

$3x + y = 8$

EXAMPLE 6

Find the slope of the line when the equation is $3x + 2y = 6$.

Solution

We can solve the equation for y in terms of x and then compare it to the slope-intercept form to determine its slope. Thus

$$3x + 2y = 6$$

$$2y = -3x + 6$$

$$y = -\frac{3}{2}x + 3$$

$$y = -\frac{3}{2}x + 3 \qquad y = mx + b$$

The slope of the line is $-\frac{3}{2}$. Furthermore, the y intercept is 3.

▼ PRACTICE YOUR SKILL

Find the slope of the line when the equation is $4x - 5y = 10$. $m = \frac{4}{5}$ ■

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EXAMPLE 7 Graph the line determined by the equation $y = \frac{2}{3}x - 1$.

Solution

Comparing the given equation to the general slope-intercept form, we see that the slope of the line is $\frac{2}{3}$ and the y intercept is -1. Because the y intercept is -1, we can plot the point $(0, -1)$. Then, because the slope is $\frac{2}{3}$, let's move 3 units to the right and 2 units up from $(0, -1)$ to locate the point $(3, 1)$. The two points $(0, -1)$ and $(3, 1)$ determine the line in Figure 3.44. (Again, you should determine a third point as a check point.)

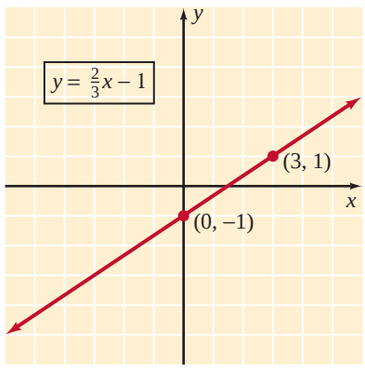

Figure 3.44

▼ **PRACTICE YOUR SKILL**

Graph the line determined by the equation $y = \frac{1}{4}x + 2$. See answer section. ■

In general, if the equation of a nonvertical line is written in slope-intercept form ($y = mx + b$), then the coefficient of x is the slope of the line and the constant term is the y intercept. (Remember that the concept of slope is not defined for a vertical line.)

We use two forms of equations of straight lines extensively. They are the **standard form** and the **slope-intercept form**, and we describe them as follows.

Standard Form. $Ax + By = C$, where B and C are integers and A is a nonnegative integer (A and B not both zero).

Slope-Intercept Form. $y = mx + b$, where m is a real number representing the slope and b is a real number representing the y intercept.

6 Find the Equations for Parallel or Perpendicular Lines

We can use two important relationships between lines and their slopes to solve certain kinds of problems. It can be shown that nonvertical parallel lines have the same slope and that two nonvertical lines are perpendicular if the product of their

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slopes is -1. (Details for verifying these facts are left to another course.) In other words, if two lines have slopes m_1 and m_2, respectively, then

1. The two lines are parallel if and only if $m_1 = m_2$.
2. The two lines are perpendicular if and only if $(m_1)(m_2) = -1$.

The following examples demonstrate the use of these properties.

EXAMPLE 8

(a) Verify that the graphs of $2x + 3y = 7$ and $4x + 6y = 11$ are parallel lines.
(b) Verify that the graphs of $8x - 12y = 3$ and $3x + 2y = 2$ are perpendicular lines.

Solution

(a) Let's change each equation to slope-intercept form.

$$2x + 3y = 7 \quad \rightarrow \quad 3y = -2x + 7$$
$$y = -\frac{2}{3}x + \frac{7}{3}$$

$$4x + 6y = 11 \quad \rightarrow \quad 6y = -4x + 11$$
$$y = -\frac{4}{6}x + \frac{11}{6}$$
$$y = -\frac{2}{3}x + \frac{11}{6}$$

Both lines have a slope of $-\frac{2}{3}$, but they have different y intercepts. Therefore, the two lines are parallel.

(b) Solving each equation for y in terms of x, we obtain

$$8x - 12y = 3 \quad \rightarrow \quad -12y = -8x + 3$$
$$y = \frac{8}{12}x - \frac{3}{12}$$
$$y = \frac{2}{3}x - \frac{1}{4}$$

$$3x + 2y = 2 \quad \rightarrow \quad 2y = -3x + 2$$
$$y = -\frac{3}{2}x + 1$$

Because $\left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1$ (the product of the two slopes is -1), the lines are perpendicular.

▼ PRACTICE YOUR SKILL

(a) Verify that the graphs of $x - 3y = 2$ and $2x - 6y = 7$ are parallel lines.
(b) Verify that the graphs of $2x - 5y = 3$ and $5x + 2y = 8$ are perpendicular lines. (a) $m_1 = m_2 = \frac{1}{3}$ (b) $m_1 = \frac{2}{5}$, $m_2 = -\frac{5}{2}$ ■

Remark: The statement "the product of two slopes is -1" is the same as saying that the two slopes are negative reciprocals of each other; that is, $m_1 = -\frac{1}{m_2}$.

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EXAMPLE 9

Find the equation of the line that contains the point (1, 4) and is parallel to the line determined by $x + 2y = 5$.

Solution

First, let's draw a figure to help in our analysis of the problem (Figure 3.45). Because the line through (1, 4) is to be parallel to the line determined by $x + 2y = 5$, it must have the same slope. Let's find the slope by changing $x + 2y = 5$ to the slope-intercept form:

$$\begin{aligned} x + 2y &= 5 \\ 2y &= -x + 5 \\ y &= -\frac{1}{2}x + \frac{5}{2} \end{aligned}$$

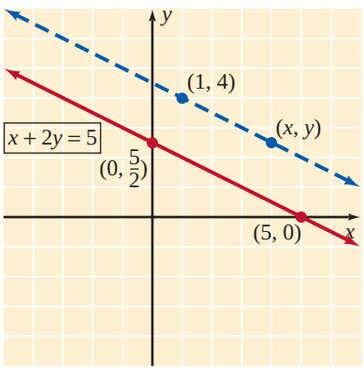

Figure 3.45

The slope of both lines is $-\frac{1}{2}$. Now we can choose a variable point (x, y) on the line through (1, 4) and proceed as we did in earlier examples.

$$\begin{aligned} \frac{y - 4}{x - 1} &= \frac{1}{-2} \\ 1(x - 1) &= -2(y - 4) \\ x - 1 &= -2y + 8 \\ x + 2y &= 9 \end{aligned}$$

▼ PRACTICE YOUR SKILL

Find the equation of the line that contains the point $(-2, 7)$ and is parallel to the line determined by $3x + y = 4$. $3x + y = 1$ ■

EXAMPLE 10

Find the equation of the line that contains the point $(-1, -2)$ and is perpendicular to the line determined by $2x - y = 6$.

Solution

First, let's draw a figure to help in our analysis of the problem (Figure 3.46). Because the line through $(-1, -2)$ is to be perpendicular to the line determined by $2x - y = 6$, its slope must be the negative reciprocal of the slope of $2x - y = 6$. Let's find the slope of $2x - y = 6$ by changing it to the slope-intercept form.

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$$2x - y = 6$$
$$-y = -2x + 6$$
$$y = 2x - 6$$

The slope is 2.

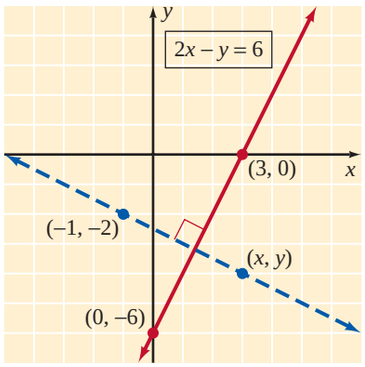

Figure 3.46

The slope of the desired line is $-\frac{1}{2}$ (the negative reciprocal of 2), and we can proceed as before by using a variable point (x, y).

$$\frac{y + 2}{x + 1} = \frac{1}{-2}$$
$$1(x + 1) = -2(y + 2)$$
$$x + 1 = -2y - 4$$
$$x + 2y = -5$$

▼ PRACTICE YOUR SKILL

Find the equation of the line that contains the point $(3, -1)$ and is perpendicular to the line determined by $5x + 2y = -10$. $2x - 5y = 11$ ■

CONCEPT QUIZ

For Problems 1–10, answer true or false.

1. If two lines have the same slope, then the lines are parallel.
2. If the slopes of two lines are reciprocals, then the lines are perpendicular.
3. In the standard form of the equation of a line $Ax + By = C$, A can be a rational number in fractional form.
4. In the slope-intercept form of an equation of a line $y = mx + b$, m is the slope.
5. In the standard form of the equation of a line $Ax + By = C$, A is the slope.
6. The slope of the line determined by the equation $3x - 2y = -4$ is $\frac{3}{2}$.
7. The concept of a slope is not defined for the line $y = 2$.
8. The concept of slope is not defined for the line $x = 2$.
9. The lines determined by the equations $x - 3y = 4$ and $2x - 6y = 11$ are parallel lines.
10. The lines determined by the equations $x - 3y = 4$ and $x + 3y = 4$ are perpendicular lines.

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Problem Set 3.4

1 Find the Equation of a Line Given a Point and a Slope

For Problems 1–8, write the equation of the line that has the indicated slope and contains the indicated point. Express final equations in standard form.

▶ **1.** $m = \frac{1}{2}$, $(3, 5)$ $x - 2y = -7$

2. $m = \frac{1}{3}$, $(2, 3)$ $x - 3y = -7$

▶ **3.** $m = 3$, $(-2, 4)$ $3x - y = -10$

4. $m = -2$, $(-1, 6)$ $2x + y = 4$

5. $m = -\frac{3}{4}$, $(-1, -3)$ $3x + 4y = -15$

6. $m = -\frac{3}{5}$, $(-2, -4)$ $3x + 5y = -26$

7. $m = \frac{5}{4}$, $(4, -2)$ $5x - 4y = 28$

8. $m = \frac{3}{2}$, $(8, -2)$ $3x - 2y = 28$

▶ **9.** x intercept of -3 and slope of $-\frac{5}{8}$ $5x + 8y = -15$

10. x intercept of 5 and slope of $-\frac{3}{10}$ $3x + 10y = 15$

2 Find the Equation of a Line Given Two Points

For Problems 11–22, write the equation of the line that contains the indicated pair of points. Express final equations in standard form.

▶**11.** $(2, 1), (6, 5)$ $x - y = 1$

12. $(-1, 2), (2, 5)$ $x - y = -3$

13. $(-2, -3), (2, 7)$ $5x - 2y = -4$

14. $(-3, -4), (1, 2)$ $3x - 2y = -1$

15. $(-3, 2), (4, 1)$ $x + 7y = 11$

▶**16.** $(-2, 5), (3, -3)$ $8x + 5y = 9$

17. $(-1, -4), (3, -6)$ $x + 2y = -9$

18. $(3, 8), (7, 2)$ $3x + 2y = 25$

19. $(0, 0), (5, 7)$ $7x - 5y = 0$

▶**20.** $(0, 0), (-5, 9)$ $9x + 5y = 0$

▶**21.** x intercept of 2 and y intercept of -4 $2x - y = 4$

22. x intercept of -1 and y intercept of -3 $3x + y = -3$

For Problems 23–28, the situations can be described by the use of linear equations in two variables. If two pairs of values are known, then we can determine the equation by using the approach used in Example 2 of this section. For each of the following, assume that the relationship can be expressed as a linear equation in two variables, and use the given information to determine the equation. Express the equation in slope-intercept form.

23. A company uses 7 pounds of fertilizer for a lawn that measures 5000 square feet and 12 pounds for a lawn that measures 10,000 square feet. Let y represent the pounds of fertilizer and x the square footage of the lawn. $y = \frac{1}{1000}x + 2$

▶**24.** A new diet fad claims that a person weighing 140 pounds should consume 1490 daily calories and that a 200-pound person should consume 1700 calories. Let y represent the calories and x the weight of the person in pounds. $y = \frac{7}{2}x + 1000$

25. Two banks on opposite corners of a town square had signs that displayed the current temperature. One bank displayed the temperature in degrees Celsius and the other in degrees Fahrenheit. A temperature of 10°C was displayed at the same time as a temperature of 50°F. On another day, a temperature of −5°C was displayed at the same time as a temperature of 23°F. Let y represent the temperature in degrees Fahrenheit and x the temperature in degrees Celsius. $y = \frac{9}{5}x + 32$

26. An accountant has a schedule of depreciation for some business equipment. The schedule shows that after 12 months the equipment is worth \$7600 and that after 20 months it is worth \$6000. Let y represent the worth and x represent the time in months. $y = -200x + 10{,}000$

27. A diabetic patient was told on a doctor visit that her HA1c reading of 6.5 corresponds to an average blood glucose level of 135. At the next checkup three months later, the patient had an HA1c reading of 6.0 and was told that it corresponds to an average blood glucose level of 120. Let y represent the HA1c reading and let x represent the average blood glucose level. $y = 30x - 60$

28. Hal purchased a 500-minute calling card for \$17.50. After he used all the minutes on that card he purchased another card from the same company at a price of \$26.25 for 750 minutes. Let y represent the cost of the card in dollars and let x represent the number of minutes. $y = 0.035x$

3 Find the Equation of a Line Given the Slope and y Intercept

For Problems 29–36, write the equation of the line that has the indicated slope (m) and y intercept (b). Express final equations in slope-intercept form.

▶**29.** $m = \frac{3}{7}$, $b = 4$ See below

30. $m = \frac{2}{9}$, $b = 6$ See below

31. $m = 2$, $b = -3$ $y = 2x - 3$

32. $m = -3$, $b = -1$ $y = -3x - 1$

33. $m = -\frac{2}{5}$, $b = 1$ See below

34. $m = -\frac{3}{7}$, $b = 4$ See below

35. $m = 0$, $b = -4$ See below

▶**36.** $m = \frac{1}{5}$, $b = 0$ See below

29. $y = \frac{3}{7}x + 4$ **30.** $y = \frac{2}{9}x + 6$ **33.** $y = -\frac{2}{5}x + 1$ **34.** $y = -\frac{3}{7}x + 4$
35. $y = 0x - 4$ **36.** $y = \frac{1}{5}x + 0$

▶Blue arrows indicate Enhanced WebAssign problems.

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4 Use the Point-Slope Form to Write Equations of Lines

For Problems 37–42, use the point-slope form to write the equation of the line that has the indicated slope and contains the indicated point. Express the final answer in standard form.

37. $m = \frac{5}{2}$, $(-3, 4)$ $5x - 2y = -23$

38. $m = \frac{2}{3}$, $(1, -4)$ $2x - 3y = 14$

39. $m = -2$, $(5, 8)$ $2x + y = 18$

40. $m = -1$, $(-6, 2)$ $x + y = -4$

41. $m = -\frac{1}{3}$, $(5, 0)$ $x + 3y = 5$

42. $m = -\frac{3}{4}$, $(0, 1)$ $3x + 4y = 4$

5 Apply the Slope-Intercept Form of an Equation

For Problems 43–48, change the equation to slope-intercept form and determine the slope and y intercept of the line.

43. $3x + y = 7$
$m = -3, b = 7$

44. $5x - y = 9$
$m = 5, b = -9$

45. $3x + 2y = 9$
See below

46. $x - 4y = 3$
See below

47. $x = 5y + 12$
See below

48. $-4x - 7y = 14$
See below

For Problems 49–56, use the slope-intercept form to graph the following lines. See answer section.

49. $y = \frac{2}{3}x - 4$

50. $y = \frac{1}{4}x + 2$

51. $y = 2x + 1$

52. $y = 3x - 1$

53. $y = -\frac{3}{2}x + 4$

54. $y = -\frac{5}{3}x + 3$

55. $y = -x + 2$

56. $y = -2x + 4$

For Problems 57–66, graph the following lines using the technique that seems most appropriate. See answer section.

57. $y = -\frac{2}{5}x - 1$

58. $y = -\frac{1}{2}x + 3$

59. $x + 2y = 5$

60. $2x - y = 7$

61. $-y = -4x + 7$

62. $3x = 2y$

63. $7y = -2x$

64. $y = -3$

65. $x = 2$

66. $y = -x$

6 Find the Equations for Parallel or Perpendicular Lines

For Problems 67–78, write the equation of the line that satisfies the given conditions. Express final equations in standard form.

67. Contains the point $(2, -4)$ and is parallel to the y axis
$x + 0y = 2$

68. Contains the point $(-3, -7)$ and is parallel to the x axis
$0x + y = -7$

69. Contains the point $(5, 6)$ and is perpendicular to the y axis
$0x + y = 6$

70. Contains the point $(-4, 7)$ and is perpendicular to the x axis $x + 0y = -4$

71. Contains the point $(1, 3)$ and is parallel to the line $x + 5y = 9$ $x + 5y = 16$

72. Contains the point $(-1, 4)$ and is parallel to the line $x - 2y = 6$ $x - 2y = -9$

73. Contains the origin and is parallel to the line $4x - 7y = 3$
$4x - 7y = 0$

74. Contains the origin and is parallel to the line $-2x - 9y = 4$ $2x + 9y = 0$

75. Contains the point $(-1, 3)$ and is perpendicular to the line $2x - y = 4$ $x + 2y = 5$

76. Contains the point $(-2, -3)$ and is perpendicular to the line $x + 4y = 6$ $4x - y = -5$

77. Contains the origin and is perpendicular to the line $-2x + 3y = 8$ $3x + 2y = 0$

78. Contains the origin and is perpendicular to the line $y = -5x$ $x - 5y = 0$

THOUGHTS INTO WORDS

79. What does it mean to say that two points determine a line?

80. How would you help a friend determine the equation of the line that is perpendicular to $x - 5y = 7$ and contains the point $(5, 4)$?

81. Explain how you would find the slope of the line $y = 4$.

45. $m = -\frac{3}{2}, b = \frac{9}{2}$ 46. $m = \frac{1}{4}, b = -\frac{3}{4}$ 47. $m = \frac{1}{5}, b = -\frac{12}{5}$ 48. $m = -\frac{4}{7}, b = -2$

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FURTHER INVESTIGATIONS

82. The equation of a line that contains the two points (x_1, y_1) and (x_2, y_2) is $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$. We often refer to this as the **two-point form** of the equation of a straight line. Use the two-point form and write the equation of the line that contains each of the indicated pairs of points. Express final equations in standard form.

(a) (1, 1) and (5, 2) $x - 4y = -3$
(b) (2, 4) and (−2, −1) $5x - 4y = -6$
(c) (−3, 5) and (3, 1) $2x + 3y = 9$
(d) (−5, 1) and (2, −7) $8x + 7y = -33$

83. Let $Ax + By = C$ and $A'x + B'y = C'$ represent two lines. Change both of these equations to slope-intercept form, and then verify each of the following properties.

(a) If $\dfrac{A}{A'} = \dfrac{B}{B'} \neq \dfrac{C}{C'}$, then the lines are parallel.

(b) If $AA' = -BB'$, then the lines are perpendicular.

84. The properties in Problem 83 provide us with another way to write the equation of a line parallel or perpendicular to a given line that contains a given point not on the line. For example, suppose that we want the equation of the line perpendicular to $3x + 4y = 6$ that contains the point (1, 2). The form $4x - 3y = k$, where k is a constant, represents a family of lines perpendicular to $3x + 4y = 6$ because we have satisfied the condition $AA' = -BB'$. Therefore, to find what specific line of the family contains (1, 2), we substitute 1 for x and 2 for y to determine k.

$$4x - 3y = k$$
$$4(1) - 3(2) = k$$
$$-2 = k$$

Thus the equation of the desired line is $4x - 3y = -2$.

Use the properties from Problem 83 to help write the equation of each of the following lines.

(a) Contains (1, 8) and is parallel to $2x + 3y = 6$ $2x + 3y = 26$
(b) Contains (−1, 4) and is parallel to $x - 2y = 4$ $x - 2y = -9$
(c) Contains (2, −7) and is perpendicular to $3x - 5y = 10$ $5x - 3y = -11$
(d) Contains (−1, −4) and is perpendicular to $2x + 5y = 12$ $5x - 2y = 3$

85. The problem of finding the perpendicular bisector of a line segment presents itself often in the study of analytic geometry. As with any problem of writing the equation of a line, you must determine the slope of the line and a point that the line passes through. A perpendicular bisector passes through the midpoint of the line segment and has a slope that is the negative reciprocal of the slope of the line segment. The problem can be solved as follows:

Find the perpendicular bisector of the line segment between the points (1, −2) and (7, 8).

The midpoint of the line segment is $\left(\dfrac{1+7}{2}, \dfrac{-2+8}{2}\right) = (4, 3)$.

The slope of the line segment is $m = \dfrac{8 - (-2)}{7 - 1} = \dfrac{10}{6} = \dfrac{5}{3}$.

Hence the perpendicular bisector will pass through the point (4, 3) and have a slope of $m = -\dfrac{3}{5}$.

$$y - 3 = -\frac{3}{5}(x - 4)$$
$$5(y - 3) = -3(x - 4)$$
$$5y - 15 = -3x + 12$$
$$3x + 5y = 27$$

Thus the equation of the perpendicular bisector of the line segment between the points (1, −2) and (7, 8) is $3x + 5y = 27$.

Find the perpendicular bisector of the line segment between the points for the following. Write the equation in standard form.

(a) (−1, 2) and (3, 0) $2x - y = 1$
(b) (6, −10) and (−4, 2) $5x - 6y = 29$
(c) (−7, −3) and (5, 9) $x + y = 2$
(d) (0, 4) and (12, −4) $3x - 2y = 18$

GRAPHING CALCULATOR ACTIVITIES

86. Predict whether each of the following pairs of equations represents parallel lines, perpendicular lines, or lines that intersect but are not perpendicular. Then graph each pair of lines to check your predictions. (The properties presented in Problem 83 should be very helpful.)

(a) $5.2x + 3.3y = 9.4$ and $5.2x + 3.3y = 12.6$
(b) $1.3x - 4.7y = 3.4$ and $1.3x - 4.7y = 11.6$
(c) $2.7x + 3.9y = 1.4$ and $2.7x - 3.9y = 8.2$
(d) $5x - 7y = 17$ and $7x + 5y = 19$
(e) $9x + 2y = 14$ and $2x + 9y = 17$
(f) $2.1x + 3.4y = 11.7$ and $3.4x - 2.1y = 17.3$
(g) $7.1x - 2.3y = 6.2$ and $2.3x + 7.1y = 9.9$
(h) $-3x + 9y = 12$ and $9x - 3y = 14$
(i) $2.6x - 5.3y = 3.4$ and $5.2x - 10.6y = 19.2$
(j) $4.8x - 5.6y = 3.4$ and $6.1x + 7.6y = 12.3$

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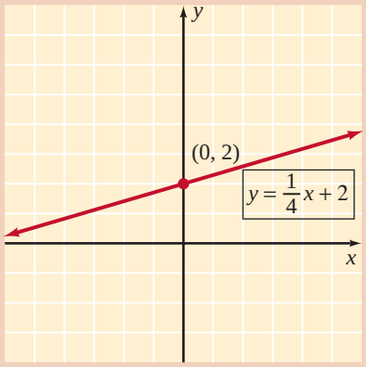
Answers to the Concept Quiz

1. True **2.** False **3.** False **4.** True **5.** False **6.** True **7.** False **8.** True **9.** True **10.** False

Answers to the Example Practice Skills

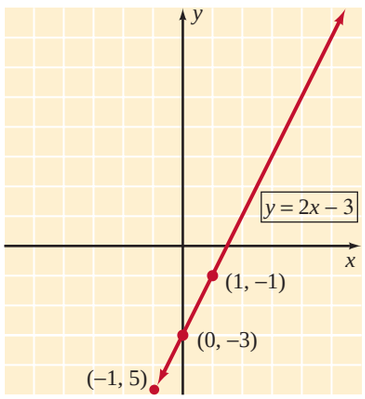
1. $3x - 4y = 13$ **2.** $5x + 6y = 40$ **3.** $3x - 2y = 8$ **4.** $4x - 3y = -23$ **5.** $3x + y = 8$ **6.** $m = \frac{4}{5}$

7.



8. **(a)** $m_1 = m_2 = \frac{1}{3}$ **(b)** $m_1 = \frac{2}{5}, m_2 = -\frac{5}{2}$ **9.** $3x + y = 1$ **10.** $2x - 5y = 11$

Chapter 3 Summary

OBJECTIVE	SUMMARY	EXAMPLE	CHAPTER REVIEW PROBLEMS
Find solutions for linear equations in two variables. (Sec. 3.1, Obj. 1, p.122)	A solution of an equation in two variables is an ordered pair of real numbers that satisfies the equation.	Find a solution for the equation $2x - 3y = -6$. **Solution** Choose an arbitrary value for x and determine the corresponding y value. Let $x = 3$; then substitute 3 for x in the equation. $2(3) - 3y = -6$ $6 - 3y = -6$ $-3y = -12$ $y = 4$ Therefore, the ordered pair (3, 4) is a solution.	Problems 1–4
Graph the solutions for linear equations. (Sec. 3.1, Obj. 3, p.125)	A graph provides a visual display of all the infinite solutions of an equation in two variables. The ordered pair solutions for a linear equation can be plotted as points on a rectangular coordinate system. Connecting the points with a straight line produces a graph of the equation.	Graph $y = 2x - 3$. **Solution** Find at least three ordered-pair solutions for the equation. We can determine that $(-1, -5)$, $(0, -3)$, and $(1, -1)$ are solutions. The graph is shown below.  	Problems 5–8

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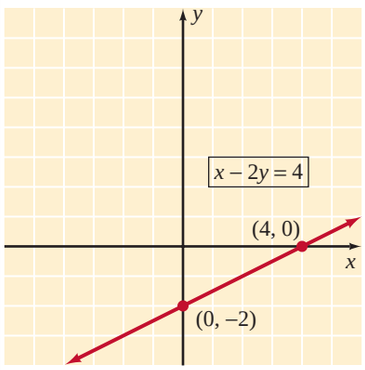
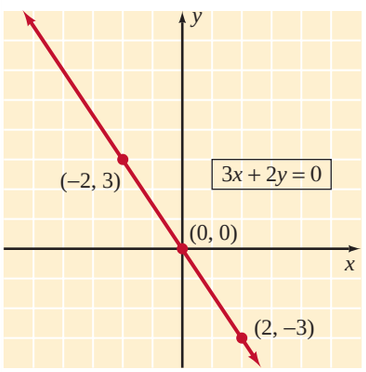
OBJECTIVE	SUMMARY	EXAMPLE	CHAPTER REVIEW PROBLEMS
Graph linear equations by finding the x and y intercepts. (Sec. 3.1, Obj. 4, p.126)	The x intercept is the x coordinate of the point where the graph intersects the x axis. The y intercept is the y coordinate of the point where the graph intersects the y axis. To find the x intercept, substitute 0 for y in the equation and then solve for x. To find the y intercept, substitute 0 for x in the equation and then solve for y. Plot the intercepts and connect them with a straight line to produce the graph.	Graph $x - 2y = 4$. **Solution** Let $y = 0$. $x - 2(0) = 4$ $x = 4$ Let $x = 0$. $0 - 2y = 4$ $y = -2$ 	Problems 9–12
Graph lines passing through the origin, vertical lines, and horizontal lines. (Sec. 3.1, Obj. 5, p.129)	The graph of any equation of the form $Ax + By = C$, where $C = 0$, is a straight line that passes through the origin. Any equation of the form $x = a$, where a is a constant, is a vertical line. Any equation of the form $y = b$, where b is a constant, is a horizontal line.	Graph $3x + 2y = 0$. **Solution** The equation indicates that the graph will be a line passing through the origin. Solving the equation for y gives us $y = -\frac{3}{2}x$. Find at least three ordered-pair solutions for the equation. We can determine that $(-2, 3)$, $(0, 0)$, and $(2, -3)$ are solutions. The graph is shown below. 	Problems 13–18

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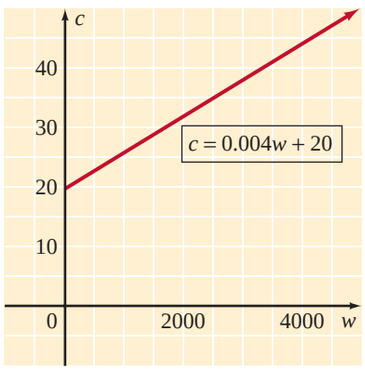
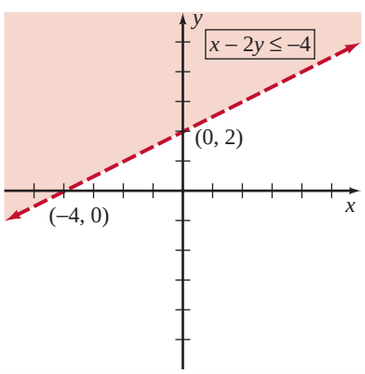
OBJECTIVE	SUMMARY	EXAMPLE	CHAPTER REVIEW PROBLEMS
Apply graphing to linear relationships. (Sec. 3.1, Obj. 6, p.130)	Many relationships between two quantities are linear relationships. Graphs of these relationships can be used to present information about the relationship.	Let c represent the cost in dollars and let w represent the gallons of water used; then the equation $c = 0.004w + 20$ can be used to determine the cost of a water bill for a household. Graph the relationship. **Solution** Label the vertical axis c and the horizontal axis w. Because of the type of application, we use only nonnegative values for w. 	Problems 19–20
Graph linear inequalities. (Sec. 3.2, Obj. 1, p.138)	To graph a linear inequality, first graph the line for the corresponding equality. Use a solid line if the equality is included in the given statement or a dashed line if the equality is not included. Then a test point is used to determine which half-plane is included in the solution set. See page 139 for the detailed steps.	Graph $x - 2y \le -4$. **Solution** First graph $x - 2y = -4$. Choose $(0, 0)$ as a test point. Substituting $(0, 0)$ into the inequality yields $0 \le -4$. Because the test point $(0, 0)$ makes the inequality a false statement, the half-plane not containing the point $(0, 0)$ is in the solution. 	Problems 21–26

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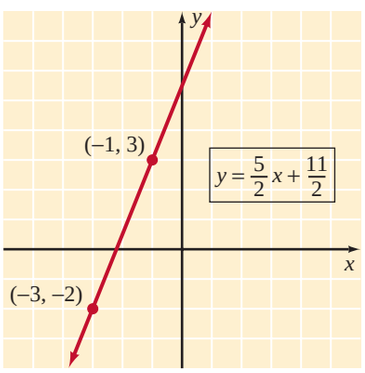
OBJECTIVE	SUMMARY	EXAMPLE	CHAPTER REVIEW PROBLEMS
Find the distance between two points. (Sec. 3.3, Obj. 1, p.143)	The distance between any two points (x_1, y_1) and (x_2, y_2) is given by the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Find the distance between $(1, -5)$ and $(4, 2)$. **Solution** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(4 - 1)^2 + (2 - (-5))^2}$ $d = \sqrt{(3)^2 + (7)^2}$ $d = \sqrt{9 + 49} = \sqrt{58}$	Problems 27–29
Find the slope of a line. (Sec. 3.3, Obj. 2, p.146)	The slope (denoted by m) of a line determined by the points (x_1, y_1) and (x_2, y_2) is given by the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_2 \neq x_1$	Find the slope of a line that contains the points $(-1, 2)$ and $(7, 8)$. **Solution** Use the slope formula: $m = \frac{8 - 2}{7 - (-1)} = \frac{6}{8} = \frac{3}{4}$ Thus the slope of the line is $\frac{3}{4}$.	Problems 30–32
Use slope to graph lines. (Sec. 3.3, Obj. 3, p.149)	A line can be graphed knowing a point on the line and the slope by plotting the point and from that point using the slope to locate another point on the line. Then those two points can be connected with a straight line to produce the graph.	Graph the line that contains the point $(-3, -2)$ and has a slope of $\frac{5}{2}$. **Solution** From the point $(-3, -2)$, locate another point by moving up 5 units and to the right 2 units to obtain the point $(-1, 3)$. Then draw the line.  	Problems 33–36

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OBJECTIVE	SUMMARY	EXAMPLE	CHAPTER REVIEW PROBLEMS
Apply slope to solve problems. (Sec. 3.3, Obj. 4, p.150)	The concept of slope is used in most situations where an incline is involved. In highway construction the word grade is used for slope.	A certain highway has a grade of 2%. How many feet does it rise in a horizontal distance of one-third of a mile (1760 feet)? **Solution** A 2% grade is equivalent to a slope of $\frac{2}{100}$. We can set up the proportion $\frac{2}{100} = \frac{y}{1760}$; then solving for y gives us $y = 35.2$. So the highway rises 35.2 feet in one-third of a mile.	Problems 37–38
Apply the slope intercept form of an equation of a line. (Sec. 3.4, Obj. 5, p.159)	The equation $y = mx + b$ is referred to as the slope-intercept form of the equation of a line. If the equation of a nonvertical line is written in this form, then the coefficient of x is the slope and the constant term is the y intercept.	Change the equation $2x + 7y = -21$ to slope-intercept form and determine the slope and y intercept. **Solution** Solve the equation $2x + 7y = -21$ for y. $2x + 7y = -21$ $7y = -2x - 21$ $y = \frac{-2}{7}x - 3$ The slope is $-\frac{2}{7}$ and the y intercept is -3.	Problems 39–41
Find the equation of a line given the slope and a point contained in the line. (Sec. 3.4, Obj. 1, p.156)	To determine the equation of a straight line given a set of conditions, we can use the point-slope form $y - y_1 = m(x - x_1)$, or $m = \frac{y - y_1}{x - x_1}$. The result can be expressed in standard form or slope-intercept form.	Find the equation of a line that contains the point $(1, -4)$ and has a slope of $\frac{3}{2}$. **Solution** Substitute $\frac{3}{2}$ for m and $(1, -4)$ for (x_1, y_1) into the formula $m = \frac{y - y_1}{x - x_1}$: $\frac{3}{2} = \frac{y - (-4)}{x - 1}$ Simplifying this equation yields $3x - 2y = 11$.	Problems 42–44

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OBJECTIVE	SUMMARY	EXAMPLE	CHAPTER REVIEW PROBLEMS
Find the equation of a line given two points contained in the line. (Sec. 3.4, Obj. 2, p.157)	First calculate the slope of the line. Substitute the slope and the coordinates of one of the points into $y - y_1 = m(x - x_1)$ or $m = \frac{y - y_1}{x - x_1}$.	Find the equation of a line that contains the points $(-3, 4)$ and $(-6, 10)$. **Solution** First calculate the slope. $m = \frac{10 - 4}{-6 - (-3)} = \frac{6}{-3} = -2$ Now substitute -2 for m and $(-3, 4)$ for (x_1, y_1) in the formula $y - y_1 = m(x - x_1)$. $y - 4 = -2(x - (-3))$ Simplifying this equation yields $2x + y = -2$.	Problems 45–46, 50–53
Find the equations for parallel and perpendicular lines. (Sec. 3.4, Obj. 6, p.161)	If two lines have slopes m_1 and m_2, respectively, then: 1. The two lines are parallel if and only if $m_1 = m_2$. 2. The two lines are perpendicular if and only if $(m_1)(m_2) = -1$.	Find the equation of a line that contains the point $(2, 1)$ and is parallel to the line $y = 3x + 4$. **Solution** The slope of the parallel line is 3. Therefore, use this slope and the point $(2, 1)$ to determine the equation: $y - 1 = 3(x - 2)$ Simplifying this equation yields $y = 3x - 5$.	Problems 47–49

Chapter 3 Review Problem Set

For Problems 1–4, determine which of the ordered pairs are solutions of the given equation.

1. $4x + y = 6$; $(1, 2), (6, 0), (-1, 10)$ $(1, 2), (-1, 10)$

2. $-x + 2y = 4$; $(-4, 1), (-4, -1), (0, 2)$ $(0, 2)$

3. $3x + 2y = 12$; $(2, 3), (-2, 9), (3, 2)$ $(2, 3), (-2, 9)$

4. $2x - 3y = -6$; $(0, -2), (-3, 0), (1, 2)$ $(-3, 0)$

For Problems 5–8, complete the table of values for the equation and graph the equation. See answer section.

5. $y = 2x - 5$

x	-1	0	1	4
y	-7	-5	-3	3

6. $y = -2x - 1$

x	-3	-1	0	2
y	5	1	-1	-5

7. $y = \frac{3x - 4}{2}$

x	-2	0	2	4
y	-5	-2	1	4

8. $2x - 3y = 3$

x	-3	0	3
y	-3	-1	1

For Problems 9–12, graph each equation by finding the x and y intercepts. See answer section.

9. $2x - y = 6$

10. $-3x - 2y = 6$

11. $x - 2y = 4$

12. $5x - y = -5$

For Problems 13–18, graph each equation. See answer section.

13. $y = -4x$

14. $2x + 3y = 0$

15. $x = 1$

16. $y = -2$

17. $y = 4$

18. $x = -3$

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19. **(a)** An apartment moving company charges according to the equation $c = 75h + 150$, where c represents the charge in dollars and h represents the number of hours for the move. Complete the following table.

h	1	2	3	4
c	225	300	375	450

(b) Labeling the horizontal axis h and the vertical axis c, graph the equation $c = 75h + 150$ for nonnegative values, of h. See answer section.

(c) Use the graph from part (b) to approximate values of c when $h = 1.5$ and 3.5.

(d) Check the accuracy of your reading from the graph in part (c) by using the equation $c = 75h + 150$.

20. **(a)** The value added tax is computed by the equation $t = 0.15v$, where t represents the tax and v represents the value of the goods. Complete the following table.

v	100	200	350	400
t	15	30	52.50	60

(b) Labeling the horizontal axis v and the vertical axis t, graph the equation $t = 0.15v$ for nonnegative values of v. See answer section.

(c) Use the graph from part (b) to approximate values of t when $v = 250$ and $v = 300$.

(d) Check the accuracy of your reading from the graph in part (c) by using the equation $t = 0.15v$.

For Problems 21–26, graph each inequality. See answer section.

21. $-x + 3y < -6$ **22.** $x + 2y \geq 4$

23. $2x - 3y \leq 6$ **24.** $y > -\frac{1}{2}x + 3$

25. $y < 2x - 5$ **26.** $y \geq \frac{2}{3}x$

27. Find the distance between each of the pairs of points.
(a) $(-1, 5)$ and $(1, -2)$ $\sqrt{53}$
(b) $(5, 0)$ and $(2, 7)$ $\sqrt{58}$

28. Find the lengths of the sides of a triangle whose vertices are at $(2, 3)$, $(5, -1)$, and $(-4, -5)$. 5, 10, $\sqrt{97}$

29. Verify that $(1, 2)$ is the midpoint of the line segment joining $(-3, -1)$ and $(5, 5)$.

30. Find the slope of the line determined by each pair of points.
(a) $(3, 4), (-2, -2)$ $\frac{6}{5}$ **(b)** $(-2, 3), (4, -1)$ $-\frac{2}{3}$

31. Find y if the line through $(-4, 3)$ and $(12, y)$ has a slope of $\frac{1}{8}$. 5

32. Find x if the line through $(x, 5)$ and $(3, -1)$ has a slope of $-\frac{3}{2}$. -1

For Problems 33–36, graph the line that has the indicated slope and contains the indicated point. See answer section.

33. $m = -\frac{1}{2}$, $(0, 3)$ **34.** $m = \frac{3}{5}$, $(0, -4)$

35. $m = 3$, $(-1, 2)$ **36.** $m = -2$, $(1, 4)$

37. A certain highway has a 6% grade. How many feet does it rise in a horizontal distance of 1 mile (5280 feet)? 316.8 ft

38. If the ratio of rise to run is to be $\frac{2}{3}$ for the steps of a staircase and the run is 12 inches, find the rise. 8 in.

39. Find the slope of each of the following lines.
(a) $4x + y = 7$ $m = -4$ **(b)** $2x - 7y = 3$ $m = \frac{2}{7}$

40. Find the slope of any line that is perpendicular to the line $-3x + 5y = 7$. $-\frac{5}{3}$

41. Find the slope of any line that is parallel to the line $4x + 5y = 10$. $-\frac{4}{5}$

For Problems 42–49, write the equation of the line that satisfies the stated conditions. Express final equations in standard form.

42. Having a slope of $-\frac{3}{7}$ and a y intercept of 4 $3x + 7y = 28$

43. Containing the point $(-1, -6)$ and having a slope of $\frac{2}{3}$ $2x - 3y = 16$

44. Containing the point $(3, -5)$ and having a slope of -1 $x + y = -2$

45. Containing the points $(-1, 2)$ and $(3, -5)$ $7x + 4y = 1$

46. Containing the points $(0, 4)$ and $(2, 6)$ $x - y = -4$

47. Containing the point $(2, 5)$ and parallel to the line $x - 2y = 4$ $x - 2y = -8$

48. Containing the point $(-2, -6)$ and perpendicular to the line $3x + 2y = 12$ $2x - 3y = 14$

49. Containing the point $(-8, 3)$ and parallel to the line $4x + y = 7$ $4x + y = -29$

50. The taxes for a primary residence can be described by a linear relationship. Find the equation for the relationship if the taxes for a home valued at \$200,000 are \$2400, and the taxes are \$3150 when the home is valued at \$250,000. Let y be the taxes and x the value of the home. Write the equation in slope-intercept form. See below

51. The freight charged by a trucking firm for a parcel under 200 pounds depends on the miles it is being shipped. To ship a 150-pound parcel 300 miles, it costs \$40. If the same parcel is shipped 1000 miles, the cost is \$180. Assume the relationship between the cost and miles is linear. Find the equation for the relationship. Let y be the cost and x be the miles. Write the equation in slope-intercept form. $y = \frac{1}{5}x - 20$

50. $y = \frac{3}{200}x - 600$

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52. On a final exam in math class, the number of points earned has a linear relationship with the number of correct answers. John got 96 points when he answered 12 questions correctly. Kimberly got 144 points when she answered 18 questions correctly. Find the equation for the relationship. Let y be the number of points and x be the number of correct answers. Write the equation in slope-intercept form. $y = 8x$

53. The time needed to install computer cables has a linear relationship with the number of feet of cable being installed. It takes $1\frac{1}{2}$ hours to install 300 feet, and 1050 feet can be installed in 4 hours. Find the equation for the relationship. Let y be the feet of cable installed and x be the time in hours. Write the equation in slope-intercept form. $y = 300x - 150$

Chapter 3 Test

1. Determine which of the ordered pairs are solutions of the equation $-2x + y = 6$: $(-1, 4), (2, 2), (4, -2), (-3, 0), (10, 26)$. $(-1, 4), (-3, 0), (10, 26)$

2. Find the slope of the line determined by the points $(-2, 4)$ and $(3, -2)$. $-\frac{6}{5}$

3. Find the slope of the line determined by the equation $3x - 7y = 12$. $m = \frac{3}{7}$

4. Find the length of the line segment whose endpoints are $(4, 2)$ and $(-3, -1)$. $\sqrt{58}$

5. What is the slope of all lines that are parallel to the line $7x - 2y = 9$? $\frac{7}{2}$

6. What is the slope of all lines that are perpendicular to the line $4x + 9y = -6$? $\frac{9}{4}$

7. The grade of a highway up a hill is 25%. How much change in horizontal distance is there if the vertical height of the hill is 120 feet? 480 ft

8. Suppose that a highway rises 200 feet in a horizontal distance of 3000 feet. Express the grade of the highway to the nearest tenth of a percent. 6.7%

9. If the ratio of rise to run is to be $\frac{3}{4}$ for the steps of a staircase and the rise is 32 centimeters, find the run to the nearest centimeter. 43 cm

10. Find the x intercept of the line $3x - y = -6$. -2

11. Find the y intercept of the line $y = \frac{3}{5}x - \frac{2}{3}$. $-\frac{2}{3}$

12. Graph the line that contains the point $(-2, 3)$ and has a slope of $-\frac{1}{4}$.
See answer section.

13. Find the x and y intercepts for the line $x - 4y = -4$ and graph the line.
See answer section.

For Problems 14–18, graph each equation.
See answer section.

14. $y = -x - 3$

15. $-3x + y = 5$

16. $3y = 2x$

17. $y = 3$

18. $y = \frac{-x - 1}{4}$

For Problems 19 and 20, graph each inequality.
See answer section.

19. $2x - y < 4$

20. $3x + 2y \geq 6$

21. Find the equation of the line that has a slope of $-\frac{3}{2}$ and contains the point $(4, -5)$. Express the equation in standard form. $3x + 2y = 2$

22. Find the equation of the line that contains the points $(-4, 2)$ and $(2, 1)$. Express the equation in slope-intercept form. $y = -\frac{1}{6}x + \frac{4}{3}$

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

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10. _____

11. _____

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23. _____

24. _____

25. _____

23. Find the equation of the line that is parallel to the line $5x + 2y = 7$ and contains the point $(-2, -4)$. Express the equation in standard form. $5x + 2y = -18$

24. Find the equation of the line that is perpendicular to the line $x - 6y = 9$ and contains the point $(4, 7)$. Express the equation in standard form. $6x + y = 31$

25. The monthly bill for a cellular phone can be described by a linear relationship. Find the equation for this relationship if the bill for 750 minutes used is \$35.00 and the bill for 550 minutes used is \$31.00. Let y represent the amount of the bill and let x represent the number of minutes used. Write the equation in slope-intercept form. $y = \frac{1}{50}x + 20$ or $y = 0.02x + 20$